Application on Jacobi Polynomials to some Nonlinear Oscillations

By

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1. Introduction

Recently ultraspherical polynomials have been used to solve some nonlinear free oscillation problems [1, 2, 3]. The author [4, 5] has applied Gegenbauer polynomials to some nonlinear forced oscillation problems. The author [6] has also applied Jacobi polynomials to the study of nonlinear free oscillations.

In the present paper the general forced oscillation problem is solved with the help of Jacobi polynomials. The self sustained oscillations have also been considered.

2. Jacobi polynomials

Jacobi polynomials $P_n^{(\alpha,\beta)}(x)$ are sets of polynomials orthogonal in the interval $(-1, 1)$ with respect to the weight factor $(1-x)^\alpha (1+x)^\beta$, each set corresponding to values of α and β such that $Re\ \alpha > -1$, $Re\ \beta > -1$. They may be obtained from [7, p. 271].

$$(2\cdot1) \qquad \sum_{n=0}^{\infty} P_n^{(\alpha,\beta)}(x)\ t^n = 2^{\alpha+\beta}\ \rho^{-1}\ (1 + t + \rho)^{-\beta}\ (1 - t + \rho)^{-\alpha},$$

where

$$(2\cdot2) \qquad \rho = (1 - 2\ x\ t + t^2)^{\frac{1}{2}}.$$

The ultraspherical (Gegenbauer), Legendre and Chebyshev polynomials are special cases of Jacobi polynomials.

In the interval $(-A, A)$ Jacobi polynomials are defined as sets of polynomials orthogonal in this interval with respect to the weight factor $(1 - x/A)^\alpha (1 + x/A)^\beta$. This gives rise to the polynomials $P_n^{(\alpha,\beta)}(x/A)$.

3. Linear Jacobi polynomial approximation

An arbitrary function restricted by very few conditions (absolute integrability suffices if Fejér's summation method is employed), can be expanded in a series of Jacobi polynomials [8, p. 451]. Thus for a function $f(x)$ expandable in terms of these polynomials in the interval $(-A, A)$, one obtains

$$(3 \cdot 1) \qquad f(x) = \sum_{n=0}^{\infty} a_n^{(\alpha, \beta)} P_n^{(\alpha, \beta)} (x/A),$$

where the coefficients $a_n^{(\alpha, \beta)}$ are given by

$$(3 \cdot 2) \qquad a_n^{(\alpha, \beta)} = \frac{\int_{-1}^{1} f(Ax) P_n^{(\alpha, \beta)} (x) (1-x)^{\alpha} (1+x)^{\beta} dx}{\int_{-1}^{1} [P_n^{(\alpha, \beta)} (x)]^2 (1-x)^{\alpha} (1+x)^{\beta} dx}$$

If the series $(3 \cdot 1)$ is truncated after the second term, one obtains a linear approximation

$$(3 \cdot 3) \qquad f_*(x) = a_0^{(\alpha, \beta)} P_0^{(\alpha, \beta)} (x/A) + a_1^{(\alpha, \beta)} P_1^{(\alpha, \beta)} (x/A),$$

where star denotes approximation.

4. The forced oscillation problem

Here we consider the forced oscillation problem characterised by the differential equation

$$(4 \cdot 1) \qquad \ddot{x} + \dot{x} g(x) + f(x) = E_0 + E_1 \cos \omega\tau, \quad \dot{x} = \frac{dx}{d\tau},$$

where either $g(x)$ or $f(x)$ or both may be nonlinear function of x.

It has been shown in [9] that if (i) $f(x)$ and $g(x)$ have first derivatives, (ii) m, n, p all positive numbers exist such that

(a) $g(x) > n > 0$ when $|x| > m$ otherwise $g(x) > -p$,

(b) $x f(x) > 0$ when $|x| > m$,

(c) $\underset{|x| \to \infty}{Lt} |f(x)| = \infty$, so that if $F(x) = \int_0^x f(x) \, dx$, $\underset{x \to \infty}{Lt} \frac{f(x)}{F(x)} = 0$,

then $(4 \cdot 1)$ has a solution of period $2\pi/\omega$.

Equation $(4 \cdot 1)$ can be written in the form

$$(4 \cdot 2) \qquad \ddot{x} + \frac{d}{d\tau}(G(x)) + f(x) \quad E_0 + E_1 \cos \omega\tau, \text{ where } G(x) = \int_0^x g(x) \, dx.$$

If γ and δ are the minimum and maximum amplitudes of the motion and if $G(x)$ and $f(x)$ are absolutely integrable functions, they can be expanded in the interval (γ, δ) in terms of shifted Jacobi polynomials $P_n^{(\alpha, \beta)} (x')$, where [10, p. 58]

$$(4 \cdot 3) \qquad x' = \frac{2x - \gamma - \delta}{\delta - \gamma}.$$

[446]

Letting $2A = \delta - \gamma$ and $x_c = \dfrac{\gamma + \delta}{2}$, (4·3) becomes

(4·4)
$$x' = \frac{x - x_c}{A}.$$

In case of symmetric oscillations, however,

$$\delta = -\gamma \text{ and } x' = x.$$

Expanding $G(x)$ and $f(x)$ in a series of Jacobi polynomials, truncating after the linear term and substituting the resulting expressions in (4·2), we get

(4·5)
$$\ddot{x} + k_* \dot{x} + \omega_*^2 x = E_0 + E_1 \cos \omega\tau - \omega_1^2, \text{ where}$$

(4·6)
$$k_* = a_1^{(\alpha,\beta)} \frac{2 + \alpha + \beta}{2A}.$$

(4·7)
$$\omega_*^2 = b_1^{(\alpha,\beta)} \frac{2 + \alpha + \beta}{2A} \qquad \text{and}$$

(4·8)
$$\omega_1^2 = b_0^{(\alpha,\beta)} + b_1^{(\alpha,\beta)} \left[\frac{\alpha - \beta}{2} - \frac{2 + \alpha + \beta}{2A} x_c \right],$$

$a_0^{(\alpha,\beta)}$, $a_1^{(\alpha,\beta)}$, $b_0^{(\alpha,\beta)}$ and $b_1^{(\alpha,\beta)}$ being the first two coefficients in the expansions of $G(x)$ and $f(x)$ respectively.

The forced oscillation corresponding to the linearised equation (4·5) is given by

(4·9)
$$x_* = \frac{E_0 - \omega_1^2}{\omega_*^2} + \frac{E_1}{[k_*^2 \omega^2 + (\omega_*^2 - \omega^2)^2]^{\frac{1}{2}}} \cos (\omega\tau + \phi),$$

where

(4·10)
$$\phi = \tan^{-1} \frac{k_*^2 \omega}{\omega_*^2 - \omega^2}$$

5. Self sustained oscillations

The present technique can also be applied to the study of free as well as forced self sustained oscillations typified by Vander Pol's equation. We shall first consider the free oscillations.

(a) *Free oscillations*

They are characterised by the Vander Pol's equation

(5·1)
$$\ddot{x} - \varepsilon (1 - x^2) \dot{x} + x = 0,$$

where ε is a parameter. They may also be represented by Rayleigh's equation

[447]

$$(5\cdot2) \qquad \ddot{y} - \varepsilon \left(\dot{y} - \frac{\dot{y}^3}{3} \right) + y = 0,$$

which is obtained through the transformation $\dot{y} = x$.

Equation $(5\cdot1)$ is a special case of $(4\cdot2)$ with $G(x) = -\varepsilon\left(x - \frac{x^3}{3}\right)$, $f(x) = x$ and $E_0 = E_1 = 0$.

Now Verma [11] has shown that

$$(5\cdot3) \qquad \int_{-1}^{1} x^s (1-x)^\alpha (1+x)^\beta P_n^{(\alpha,\beta)}(x)\, dx$$

$$= \frac{2^{n+\alpha+\beta+1}\, s!\, \Gamma(1+\alpha+n)\, \Gamma(1+\beta+s)}{n!\,(s-n)!\,\Gamma(n+s+\alpha+\beta+2)} \; {}_2F_1 \left[\begin{matrix} -s+n,\ 1+\alpha+n; \\ -\beta-s; \end{matrix} \; -1 \right],\ s > n.$$

We also have the result [7, p. 261]

$$(5\cdot4) \qquad \int_{-1}^{1} x^s (1-x)^\alpha (1+x)^\beta P_n^{(\alpha,\beta)}(x)\, dx = 0,\ s < n,$$

and Bhonsle [12, p. 160] has shown that

$$(5\cdot5) \qquad \int_{-1}^{1} x^n (1-x)^\alpha (1+x)^\beta P_n^{(\alpha,\beta)}(x)\, dx = \frac{2^{1+\alpha+\beta+n}\,\Gamma(1+\alpha+n)\,\Gamma(1+\beta+n)}{\Gamma(2+\alpha+\beta+n)}.$$

Approximating $\varepsilon(x - x^3/3)$ by means of linear Jacobi polynomials using equations $(3\cdot2)$, $(3\cdot3)$ and $(5\cdot3)$ to $(5\cdot5)$, one obtains

$$(5\cdot6) \qquad \{ \varepsilon(x - x^3/3) \}_* = k_*\, x + \frac{(\alpha-\beta)A}{2+\alpha+\beta}\, [\, k_* - \omega_1^2 \,]\ ,\ \text{where}$$

A is the amplitude of the symmetric oscillation

$$(5\cdot7) \qquad k_* = \varepsilon \left[1 - \frac{A^2\{(\alpha-\beta)^2 + \alpha + \beta + 4\}}{(\alpha+\beta+4)(\alpha+\beta+5)} \right] \text{and}$$

$$(5\cdot8) \qquad \omega_1^2 = \varepsilon \left[1 - \frac{A^2\{(\alpha-\beta)^2 + 3(\alpha+\beta) + 8\}}{3(\alpha+\beta+3)(\alpha+\beta+4)} \right].$$

Linearising $(5\cdot1)$ with the help of $(5\cdot6)$, we get

$$(5\cdot9) \qquad \ddot{x} - k_*\, \dot{x} + x = 0.$$

Now for a steady state periodic motion to exist $k_* = 0$, which yields the condition on maximum amplitude of the periodic motion corresponding to a limit cycle. We then have

$$(5\cdot10) \qquad 1 - \frac{(A^2\{(\alpha-\beta)^2 + \alpha + \beta + 4\}}{(\alpha+\beta+4)(\alpha+\beta+5)} = 0. \qquad -$$

If $\alpha = \beta = -\frac{1}{2}$, $(5\cdot10)$ yields $A = 2$, which is a well-known result.

(b) Forced oscillations

Vander Pol's equation with a forcing term : We note that the linearisation scheme when applied to the equation

(5·11)
$$\ddot{x} - \varepsilon (1 - x^2) \dot{x} + \omega_0^2 x = E \cos(\omega\tau + \phi),$$

where ε, ω_0^2 and E are positive constants, yields the steady state solution

(5·12)
$$x = \frac{E \cos(\omega\tau + \phi + \delta)}{\left[(\omega_0^2 - \omega^2)^2 + \omega^2\varepsilon^2 \left\{ 1 - \frac{(\alpha - \beta)^2 + \alpha + \beta + 4}{(\alpha + \beta + 4)(\alpha + \beta + 5)} A^2 \right\}^2 \right]},$$

where

(5·13)
$$\delta = \tan^{-1} \frac{\varepsilon \left\{ 1 - \frac{(\alpha - \beta)^2 + \alpha + \beta + 4}{(\alpha + \beta + 4)(\alpha + \beta + 5)} A^2 \right\}}{\omega_0^2 - \omega^2} = 0 \text{ as } k_* = 0.$$

The steady state amplitude is thus given by

(5·14)
$$A^2 = E^2 \left[(\omega_0^2 - \omega^2)^2 + \omega^2\varepsilon^2 \left\{ 1 - \frac{(\alpha - {}^2) + \alpha + \beta + 4}{(\alpha + \beta + 4)(\alpha + \beta + 5)} A^2 \right\}^2 \right]^{-1}.$$

Putting $\dfrac{\omega^2 - \omega_0^2}{\omega\varepsilon} = x$,

$$\frac{(\alpha - \beta)^2 + \alpha + \beta + 4}{(\alpha + \beta + 4)(\alpha + \beta + 5)} A^2 = y \text{ and}$$

$$\frac{E^2}{\omega^2\varepsilon^2} \frac{(\alpha - \beta)^2 + \alpha + \beta + 4}{(\alpha + \beta + 4)(\alpha + \beta + 5)} = F, \text{ (5·14) becomes}$$

(5·15)
$$x^2 y + (1 - y)^2 y = F$$

For $\alpha = \beta = -\tfrac{1}{2}$ this response relation is the same as obtained by other methods in [13, p. 82] and [14, p. 155]. The technique can also be used when the restoring force is non-linear. In this case only the value of ω_0^2 changes.

Discussion and conclusions

The results obtained by the present method agree with those obtained by other methods in case of self sustained oscillations as pointed above.

The linearisation of the nonlinear differential equation governing oscillations has been accomplished by linearising the nonlinear functions (damping and restoring) by means of linear Jacobi polynomials. The results of ultraspherical (Gegenbauer) polynomial approximation can be obtained from the corresponding results obtained in this paper by putting $\alpha = \beta = \lambda - \tfrac{1}{2}$. The differential equations discussed occur in several physical problem.

[449]

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Photocatalytic synthesis of aminoacids

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Abstract

Photosynthesis of aminoacids has been obtained by exposing a mixture of glucose and ammonia in the presence of an oxidising agent like H_2O_2 or potassium persulphate. Molybdic acid is a better catalyst than vanadium pentoxide. The synthesis of aminoacids is highly facilitated by the absorption of light and the addition of phosphates. Along with photosynthesis of aminoacids, photolysis is also taking place.

Introduction

Pavlovskaya and coworkers[1] observed the formation of aminoacids by the action of ultraviolet on the solution of formaldehyde and ammonium salts in the presence of adsorbents. Deschreider[2] obtained aminoacids by exposing to ultraviolet rays a mixture containing succinic acid, maleic acid or propionic acid and ammonia, ammonium carbonate or ammonium cyanate. Miller[3,4,5] observed the synthesis of aminoacids by passing electric discharge in an atmosphere of mixture of gases such as hydrogen, methane and ammonia.

Recently Ranganayaki and Bahadur[6] have investigated the possibility of nitrogen fixation without the help of bacteria and subsequent utilization of the fixed nitrogen in the formation of aminoacids in an aqueous mixture containing para formaldehyde as the source of carbon and colloidal molybdenum oxide as catalyst in a sterile set by exposing the solution to light of a 500 watt bulb.

Formation of complex organic compounds by the action of high energy source like ultraviolet rays, X-rays and electric discharge on the mixture of simple gases like methane, ammonia, hydrogen and water, has been demonstrated by many workers like Horowitz and Miller[7]. Miller and Urey[8], Lowe *et al*[9] and others[10,11].

In the present study the photosynthesis of aminoacids has been investigated by exposing mixtures of glucose and ammonia in presence of an oxidising agent like H_2O_2 or potassium persulphate. The influence of phosphates, molybdic acid and vanadium pentoxide has also been studied.

Experimental

Sterilized sets were used in these experiments. 10 ml. of M/4 glucose solution and 10 ml of M/3 NH_3 and 0·2 gm titania were taken in six small conical flasks. 10 ml of M/40 H_2O_2 were added to all the flasks. 0·01 gm of molybdic acid or vanadium pentoxide was added in one flask each and the mixture of two in two flasks. Two sets were phosphated with 0 1 gm $CaHPO_4.2H_2O$ (one with MoO_3 + V_2O_5) and one without it) . Similar six flasks were prepared for dark set. All

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the sets were exposed to 500 watt electric bulb, the dark sets were covered with thick black cloth. The contents of the flask were analysed for aminoacids after definite intervals of time. The qualitative detection was done by paper chromatography while the quantitative estimations were carried out with respect to standard solution colorimetrically[12] after desalting. A similar set was prepared with potassium persulphate instead of H_2O_2 and aminoacids analysed after a definite interval of time.

Sterilization of sets : Sterilization of the NH_3–H_2O_2- organic compound-titania or other catalysts-phosphate systems were carried out in the following way. Requisite amounts of glucose solution, photosensitiser and phosphate were taken in various flasks. The flasks were cotton plugged and sterilised in an autoclave under 15 lb pressure for 20 minutes. Requisite amounts of NH_3, H_2O_2 or persulphate solution were then introduced by means of a sterilized pipette under aseptic conditions.

The following abbreviations for the aminoacids studied have beeen used :

Glycine . ..... Gly	Glutamic acid ....Glut
Alanine ..... Al	Serine ....Se
Valine . .....Val	Argenine .. ..Ar
Leucine..... ..Leu	Methionine............Me
Aspartic acid...Asp.	Lysine......Ly, Threonine .. ..Th

TABLE 1

10 ml. M/4 glucose + 10 ml. M/3 NH_3 + 10 ml. M/40 H_2O_2 + 0·2 gm TiO_2

Period of exposure in days	Aminoacids detected chromatographically	Amount of aminoacids with respect to glycine in gm/litre	Aminoacids detected chromatographically	Amount of aminoacids with respect to glycine in gm/litre
	LIGHT		DARK	
1	Gly.,	0·0278	...	...
2	Gly., Al.,	0·0734	Gly.,	0·0247
3	Gly., Al.,	0·0746	Al.,	0·0231
4	Gly., Al.,	0·0463	...	...
5	Gly.,	0·0342	...	...

TABLE 2

10 ml. M/4 glucose + 10 ml. M/3 NH_3 + 10 ml. M/40 H_2O_2 + 0·2 gm TiO_2 + 0·1 gm $CaHPO_4 \cdot 2H_2O$

1	Gly., Al.,	0·0316	...	...
2	Gly., Val., Asp., Al.,	0·0868	Gly., Al.,	0·0276
3	Asp., Al., Th., Gly., Val.,	0·0923	Gly., Al., Val.,	0·0291
4	Gly., Val , Asp.,	0·0526	Gly., Al.,	0·0273
5	Gly., Val.,	0·0368	Gly., (faint)	traces

TABLE 3

10 ml. M/4 glucose + 10 ml. M/3 NH$_3$ + 10 ml. M/40 H$_2$O$_2$ + 0·2 gm TiO$_2$ + 0·01 gm Molybdic acid

Period of exposure in days	Aminoacids detected chromato·graphically	Amount of amino-acids with respect to glycine in gm/litre	Aminoacids detected chromato-graphically	Amount of amino-acids with respect to glycine in gm/litre
	LIGHT		DARK	
1	Gly., Val., Al.,	0·0294	...	...
2	Gly., Val., Al., Glut.	0·0758	Gly., Al.,	0·0263
3	Gly., Val., Ly., Th., Asp.,	0·0793	Gly., Al.,	0·0242
4	Gly., Val., Ly.,	0·0489	faint	..
5	Al., Gly., Ly.,	0·0358	...	...

TABLE 4

10 ml. M/4 glucose + 10 ml. M/3 NH$_3$ + 10 ml. M/40 H$_2$O$_2$ + 0·2 gm TiO$_2$ + 0·01 gm V$_2$O$_5$

1	Gly., Val., Al.,	0·0289	...	...
2	Gly., Val., Glut.,	0·0747	Gly., Al.,	0·0258
3	Gly., Val., Ly., Asp·,	0·0762	Gly., Al.,	0·0237
4	Gly., Val., Al.,	0·0479	Gly., (faint)	...
5	Gly., Val.,	0·0351	...	...

TABLE 5

10 ml. M/4 glucose + 10 ml. M/3 NH$_3$ + 10 ml. M/40 H$_2$O$_2$ + 0·2 gm TiO$_2$ + 0·02 gm (MoO$_3$ + V$_2$O$_5$) + 0·1 gm CaHPO$_4$·2H$_2$O

1	Gly., Val., Al., Se.,	0·0367	..	
2	Gly., Val., Se., Ly., Th.,	0·0892	Gly., Val., Al.,	0·0296
3	Gly., Val., Al., Se., Th., Asp.,	0·0956	Gly., Val., Al., Ly., (faint)	0·0319
4	Gly., Val., Al., Asp.,	0·0583	Gly., Val., Al.,	0·0304
5	Asp., Gly., Al.,	0·0407	Gly., Val.,	0·0274

TABLE 6

10 ml. M/4 glucose + 10 ml. M/3 NH$_3$ + 10 ml. M/40 H$_2$O$_2$ + 0·2 gm TiO$_2$ + 0·02 gm (MoO$_3$ + V$_2$O$_5$)

1	Gly., Val., Al.,	0·0314	...	...
2	Gly., Val., Al., Glut.	0·0782	Gly., Al.,	0·0278
3	Al., Gly., Val., Th., Ly.,	0·0838	Gly., Val., Al.,	0·0259
4	Gly., Val., Ly.,	0·0503	Gly.,	0·0224
5	Gly., Al.,	0·0362	Gly., (faint)	

TABLE 7

10 ml. $M/4$ glucose + 10 ml. $M/3$ NH_3 + 10 ml. $M/40$ Potassium persulphate + 0·2 gm TiO_2

Period of exposure in days	Aminoacids detected chromatographically	Amount of aminoacids with respect to glycine in gm/litre	Aminoacids detected chromatographically	Amount of aminoacids with respect to glycine in gm/litre
	LIGHT		DARK	
1	Gly.,	0·0237	...	..
2	Val., Gly.,	0·0672	.	.
3	Gly., Val., Al., Asp.,	0·0681	Al., Val.,	0·0238
4	Gly., Val.,	0·0383	Gly.,	0·0190
5	Al.,	...	..	...

TABLE 8

10 ml. $M/4$ glucose + 10 ml. $M/3$ NH_3 + 10 ml. $M/40$ potassium persulphate + 0·2 gm TiO_2 + 0·1 gm $CaHPO_4. 2H_2O$

1	Gly., Val.,	0·0273	...	...
2	Gly., Val., Al., Asp., Ly.,	0·0836	...	..
3	Gly., Val., Ly.,	0·0731	Gly., Val.,	0·0248
4	Gly., Val.,	0·0386	Al., Val.,	0·0269
5	...	...	..	..

TABLE 9

10 ml. $M/4$ glucose + 10 ml. $M/3$ NH_3 + 10 ml. $M/40$ potassium persulphate + 0·2 gm TiO_2 + 0·01 gm MoO_3

1	Gly.,	0·0256	...	...
2	Gly., Val.,	0·0689	..	..
3	Gly., Al., Val.,	0·0713	Gly., Val.,	0·0254
4	Gly., Al ,	0·0462	Gly.,	0·022ð
5	Gly., Al.,	0·0298	...	...

TABLE 10

10 ml. $M/4$ glucose + 10 ml. $M/3$ NH_3 + 10 ml. $M/40$ potassium persulphate + 0·2 gm TiO_2 + 0·01 gm V_2O_5

1	Gly.,	0·0242	...	...
2	Gly., Val.,	0·0686	...	...
3	Gly., Val., Al.,	0·0702	Gly., Val.,	0·0242
4	Gly., Val., Al.,	0·0418	Gly.,	...
5	Gly., Al.,	0·0210	...	...

TABLE 11

10 ml. M/4 glucose + 10 ml. M/3 NH$_3$ + 10 ml. M/40 potassium persulphate + 0·2 gm TiO$_2$ + 0·02 gm (MoO$_3$ + V$_2$O$_5$)

Period of exposure in days	Aminoacids detected chromatographically	Amount of aminoacids with respect to glycine in gm/litre	Aminoacids detected chromatographically	Amount of aminoacids with respect to glycine in gm/litre
	LIGHT		DARK	
1	Gly., Val.,	0·0278	...	...
2	Gly., Val., Al.,	0·0743	...	...
3	Al., Ly., Val., Asp.,	0·0768	Gly., Val.,	0·0259
4	Al., Asp., Ly., Val.,	0·0479	Al., Val.,	0·0239
5	Val., Al.,	0·0343	...	...

TABLE 12

10 ml. M/4 glucose + 10 ml. M/3 NH$_3$ + 10 ml. M/40 potassium persulphate + 0·2 gm TiO$_2$ + 0·02 gm MoO$_3$ + V$_2$O$_5$ + 0·1 gm CaHPO$_4$.2H$_2$O

1	Gly., Val.,	0·0328	...	...
2	Gly., Val., Al., Ly., A p ,	0 0783	Gly., Al.,	0·0272
3	Gly., Al., Val., Asp.,	0·0878	Gly., Val., Al.,	0·0283
4	Gly., Val., Al.,	0·0672	Al., Gly.,	0·0267
5	Val.,	0·0386	...	...

Discussion

A perusal of the experimental results (table 1–12) shows that when the systems consisting solution of ammonia and glucose and an oxidising agent like H$_2$O$_2$ or potassium persulphate are exposed to light in the presence of titania and other photocatalysts MoO$_3$ or V$_2$O$_5$, an appreciable amounts of aminoacids are formed. The quantity as well as the number of aminoacids produced increases when the systems are phosphated with dicalcium phosphate. It is clear from the results that H$_2$O$_2$ gives better results than potassium persulphate. This may be due to the fact that potassium persulphate produces sulphuric acid after sometime which is harmful. It is interesting to note that the phosphated sets contain more aminoacids than the unphosphated ones. The reason is that the phosphates form stable complexes with proteins and aminoacids, thus making the protein or amino-acids molecules much more stable towards decomposition.

It is further observed that in systems containing molybdic acid give better results than containing V$_2$O$_5$ and the mixture of these two photocatalysts gives an appreciable increase, which is more pronounced when the systems are phosphated. This may be probably due to the difference in their photocatalytic activity.

The synthesis of aminoacids in sterile set is highly facilitated by the absorption of light. In all the cases the amounts of aminoacids formed are much higher than in the dark. In the exposed sets it seems that the energy liberated by the oxidation of the energy material and the partial conversion of NH$_3$ into nitrite

and then to nitrate as well as the energy absorbed in the form of light are utilized in the formation of aminoacids. But in the covered sets, only small amounts of aminoacids are synthesised because only the thermo-chemical energy due to the oxidation of the organic compound and the conversion of NH_3 into nitrite and nitrate is available.

It is interesting to note that aminoacids photosynthesised constantly undergo decomposition and ammonification aided by light and the addition of $CaHPO_4$. $2H_2O$ checks the decomposition of these aminoacids by forming stable phosphorylated compound with the result that the amount of aminoacids appear to be larger in solutions which are phosphated.

Pavlovskaya et al.[1] observed the formation of aminoacids by the action of ultraviolet light on the solution of formaldehyde and ammonium salts in the presence of adsorbents. Bahadur and Ranganayaki[13] observed that the formation of aminoacids in water containing dissolved CO_2 and colloidal MoO_2.

In all these systems it is observed that there is an increase in the amount of aminoacids synthesised in the beginning which is followed by a decrease on the prolonged exposure. It seems that along with photosynthesis of the aminoacids, photolysis is also taking place. Vaidyanathan, Kalyankar and Giri[14] observed the photolysis of aminoacids in presence of different sensitisers.

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Nitrogen Fixation with Nitrogen free Oxide Surfaces

By

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For nitrogen fixation it is not necessary to have soil or sand as medium. Dhar and Seshacharyulu[1] have obtained considerable nitrogen fixation on adding energy materials to chemically pure surfaces, like oxides of metals, at ordinary temperature.

For throwing light an this problem, experiments were undertaken using chemically pure surfaces like oxides of different metals instead of soil or sand mixed with energy materials. The diff rent oxides which have been used are Ferric oxide, Zinc oxide and titania. In order to study the influence of phosphates, Tata basic slag and Trichinopoly rock phosphate were used. The organic materials used are Glucose, Wheat straw and sawdust.

Experimental

Pure samples (B. D. H. products) of ferric oxide, Zinc oxide and titania were taken, washed thoroughly and dried at room temperature. 100 gms of each sample were taken in clean dishes and energy materials were added to the extent of 0.8% carbon with and without phosphate sources containing 0.5% P_2O_5.

The results obtained are recorded in the succeeding pages.

TABLE 1

100 gms of Zinc oxide + 0.8% C as Glucose

Period of Exposure in days	Total C Un-oxidised %	Total C Oxidised %	Total Nitrogen %	Increase in Nitrogen %	Efficiency (amount of N fixed in mgm/gm. of carbon oxidised)	Total available P_2O_5 %
		Light				
0	0.7843	...	...	...	...	...
30	0.6049	0.1794	0.0072	0.0072	40.1	...
60	0.5381	0.2462	0.0101	0.0101	41.0	...
90	0.5023	0.2820	0.0117	0.0117	41.4	...
120	0.4792	0.3051	0.0124	0.0124	40.6	...
		Dark				
0	0.7843	...	.	..	...	..
30	0.6417	0.1426	0.0029	0.0029	20.3	...
60	0.5805	0.2038	0.0044	0.0044	21.5	..
90	0.5489	0.2354	0.0052	0.0052	22.1	...
120	0.5247	0.2596	0.0055	0.0055	21.1	..

TABLE 2

100 gms of Zinc oxide + 0·8% C as Glucose + 0·5% P_2O_5 as Tata basic slag

Period of Exposure in days	Total C Un-oxidised %	Total C Oxidised %	Total Nitrogen %	Increase in nitrogen %	Efficiency (amount of N fixed in mgm./gm of carbon oxidised)	Total available P_2O_5 %
Light						
0	0·7368	...	...	...	...	0·2316
30	0·5244	0·2124	0·0110	0·0110	51·7	0·2572
60	0·4566	0·2802	0·0148	0·0148	52·8	0·2711
90	0·4229	0·3139	0·0167	0·0167	53·2	0·2767
120	0·3977	0·3391	0·0179	0·0179	52·7	0·2795
Dark						
0	0·7368	...		...	...	0·2316
30	0·5540	0·1828	0·0048	0·0048	26·2	0·2478
60	0·4912	0·2456	0·0068	0·0068	27·6	0·2601
90	0·4593	0·2775	0·0078	0·0078	28·1	0·2619
120	0·4307	0·3061	0·0085	0·0085	27·7	0·2671

TABLE 3

100 gms. of Zinc oxide + 0·8% C as Glucose + 0·5% P_2O_5 as Trichinopoly rock phosphate

Light						
0	0·7707	...		...	...	0·0266
30	0·5676	0·2031	0·0094	0·0094	46·2	0·0421
60	0·5005	0·2702	0·0127	0·0127	47·0	0·0506
90	0·4676	0·3031	0·0144	0·0144	47·5	0·0562
120	0·4383	0·3324	0·0156	0·0156	46·9	0·0603
Dark						
0	0·7707	...	..	...	...	0·0266
30	0·6019	0·1688	0·0040	0·0040	23·7	0·0385
60	0·5416	0·2291	0·0057	0·0057	24·8	0·0456
90	0·5059	0·2648	0·0067	0·0067	25·3	0·0498
120	0·4780	0·2927	0·0073	0·0073	24·9	0·0531

TABLE 4

100 gms. of Ferric oxide + 0·8% C as Glucose

Light						
0	0·7843	...	...	...	...	...
30	0·6140	0·1703	0·0064	0·0064	37·5	...
60	0·5458	0·2385	0·0092	0·0092	38·5	...
90	0·5098	0·2745	0·0107	0·0107	38·9	...
120	0·4867	0·2976	0·0113	0·0113	37·9	...
Dark						
0	0·7843	...		...	...	...
30	0·6496	0·1347	0·0026	0·0026	19·3	...
60	0·5874	0·1969	0·0041	0·0041	20·8	...
90	0·5553	0·2290	0·0049	0·0049	21·3	...
120	0·5304	0·2539	0·0052	0·0052	20·4	...

TABLE 5

100 gms of Ferric oxide + 0·8% C as Glucose + 0·5% P_2O_5 as Tata basic slag

Period of Exposure in days	Total C Un-oxidised %	Total C Oxidised %	Total Nitrogen %	Increase in nitrogen %	Efficiency (amount of N fixed in mgm/gm of carbon oxidised)	Total available P_2O_5 %
			Light			
0	0·7368	...	...	...	..	0·2316
30	0·5336	0·2032	0·0101	0·0101	49·7	0·2524
60	0 4667	0·2701	0·0138	0·0138	51·0	0·2618
90	0·4328	0·3040	0·0157	0·0157	51·6	0·2669
120	0·4069	0·3299	0·0168	0·0168	50·9	0·2697
			Dark			
0	0·7368	...	...	...	...	0·2316
30	0·5627	0·1741	0·0044	0·0044	25·2	0·2431
60	0·5009	0·2359	0·0063	0·0063	26·7	0·2528
90	0·4687	0·2681	0·0073	0 0073	27·2	0·2583
120	0·4403	0·2965	0·0079	0·0079	26·6	0·2614

TABLE 6

100 gms of Ferric oxide + 0·8% C as Glucose + 0·5% P_2O_5 as Trichinopoly rock phosphate

			Light			
0	0·7707	...	..	...	...	0·0266
30	0·5765	0·1942	0·0086	0·0086	44·2	0·409
60	0·5098	0·2602	0·0118	0·0118	45·2	0·0477
90	0·4762	0·2945	0·0135	0·0135	45·8	0·0523
120	0·4481	0·3226	0·0145	0·0145	44·9	0·0559
			Dark			
0	0·7707	...	...	...	...	0·0266
30	0·6106	0·1601	0 0036	0·0036	22·4	0·0374
60	0·5503	0·2204	0·0052	0·0052	23·5	0·0432
90	0·5154	0·2553	0·0062	0·0062	24·7	0·0465
120	0·4867	0·2840	0·0068	0·0068	23·9	0·0491

TABLE 7

100 gms of Titania + 0·8% C as Glucose

			Light			
0	0·7843	...	...	...	...	...
30	0·6015	0·1828	0·0075	0·0075	41·1	...
60	0·5342	0·2501	0·0105	0·0105	41·9	...
90	0·4988	0·2855	0·0121	0·0121	42·3	...
120	0·4764	0·3079	0·0128	0·0128	41·5	...
			Dark			
0	0·7843	...	...	...	...	...
30	0·6393	0·1450	0·0031	0·0031	21·3	...
60	0·5779	0·2064	0·0047	0·0047	22·7	...
90	0·5465	0·2378	0·0055	0·0055	23·1	...
120	0·5215	0·2628	0·0059	0·0059	22·4	...

TABLE 8

100 gms of Titania + 0·8% C as Glucose + 0·5% P_2O_5 as Tata basic slag

Period of Exposure in days	Total C Un-oxidised %	Total C Oxidised %	Total Nitrogen %	Increase in nitrogen %	Efficiency (amount of N fixed in mgm/gm of carbon oxidised)	Total available P_2O_5 %
			Light			
0	0·7368	.		...	...	0·2316
30	0·5210	0·2158	0·0114	0·0114	52·8	0·2595
60	0·4529	0·2839	0·0152	0·0152	53·5	0·2746
90	0·4188	0·3180	0·0172	0·0172	54·0	0·2808
120	0·3939	0·3429	0·0184	0·0184	53·6	0·2843
			Dark			
0	0·7368	...		...	...	0·2316
30	0·5512	0·1856	0·0050	0·0050	26·9	0·2487
60	0·4877	0·2491	0·0070	0·0070	28·1	0·2618
90	0·4563	0·2805	0·0081	0·0081	28·8	0·2677
120	0·4273	0·3095	0·0088	0·0088	28·4	0·2708

TABLE 9

100 gms of Titania+ 0·8% C as Glucose +0·5% P_2O_5 as Trichinopoly rock phosphate

Period of Exposure in days	Total C Un-oxidised %	Total C Oxidised %	Total Nitrogen %	Increase in nitrogen %	Efficiency (amount of N fixed in mgm/gm of carbon oxidised)	Total available P_2O_5 %
			Light			
0	0·7707	...	...	...	...	2·0266
30	0·5644	0·2063	0·0098	0·0098	47·5	0·0432
60	0·4969	0·2738	0·0132	0·0132	48·2	0·0523
90	0·4639	0·3068	0·0150	0·0150	48·8	0·0584
120	0·4349	0·3358	0·0162	0·0162	48·2	0·0632
			Dark			
0	0·7707	...	..	...	...	0·0266
30	0·5983	0·1714	0·0042	0·0042	24·5	0·0397
60	0·5390	0·2317	0·0059	0·0059	25·4	0·0471
90	0·5030	0·2677	0·0070	0·0070	26·1	0·0522
120	0·4753	0·2954	0·0076	0·0076	25·7	0·0556

TABLE 10

100 gms of Zinc oxide + 0·8% C as Wheat straw

Period of Exposure in days	Total C Un-oxidised %	Total C Oxidised %	Total Nitrogen %	Increase in nitrogen %	Efficiency (amount of N fixed in mgm/gm of carbon oxidised)	Total available P_2O_5 %
			Light			
0	0·7837	...	0·0126	...	...	...
60	0·5950	0·1887	0·0206	0·0080	42·3	...
120	0·5346	0·2491	0·0233	0·0107	42·9	...
180	0·4964	0·2873	0·0247	0·0121	42·1	...
			Dark			
0	0·7837	...	0·0126	...	...	...
60	0·5950	0·1557	0·0160	0·0034	21·8	...
120	0·5346	0·2269	0·0178	0·0052	22·9	...
180	0·4964	0·2578	0·0183	0·0057	22·1	...

TABLE 11

100 gms of Zinc oxide + 0·8% C as Wheat straw + 0·5% P_2O_5 as Tata basic slag

Period of Exposure in days	Total C Un-oxidised %	Total C oxidised %	Total Nitrogen %	Increase in Nitrogen %	Efficiency (amount of N fixed in mgm/gm of carbon oxidised)	Total available P_2O_5 %
Light						
0	0·7362	...	0·0118	...	...	0·2314
60	0·5144	0·2218	0·0237	0·0119	53·6	0·2523
120	0·5516	0·2846	0·0272	0·0154	54·1	0·2648
180	0·4204	0·3158	0.0287	0·0169	53·3	0·2724
Dark						
0	0·7362	...	0·0118	..	...	0·2314
60	0·5459	0·1903	0·0170	0·0052	27·3	0·2445
120	0·4831	0·2531	0·0189	0·0071	28·0	0·2551
180	0·4488	0·2874	0·0197	0·0079	27·4	0·2632

TABLE 12

100 gms of Zinc oxide + 0·8% C as Wheat straw + 0·5% P_2O_5 as Trichinopoly rock phosphate

Period of Exposure in days	Total C Un-oxidised %	Total C oxidised %	Total Nitrogen %	Increase in Nitrogen %	Efficiency (amount of N fixed in mgm/gm of carbon oxidised)	Total available P_2O_5 %
Light						
0	0·7701	...	0·0124	...	...	0·0266
60	0·5558	0·2143	0·0228	0·0104	48·5	0·0401
120	0·4930	0·2771	0·0260	0·0136	49·0	0·0478
180	0·4580	0·3121	0·0275	0·0151	48·3	0·0523
Dark						
0	0·7701	...	0·0124	...	...	0·0266
60	0·5889	0·1812	0·0169	0·0045	24·8	0·0362
120	0·5273	0·2428	0·0187	0·0063	25·9	0·0427
180	0·4910	0·2791	0·0195	C·0071	25·4	0·0471

TABLE 13

100 gms of Ferric oxide + 0·8% C as Wheat straw

Period of Exposure in days	Total C Un-oxidised %	Total C oxidised %	Total Nitrogen %	Increase in Nitrogen %	Efficiency (amount of N fixed in mgm/gm of carbon oxidised)	Total available P_2O_5 %
Light						
0	0·7837	...	0·0126	...	...	..
60	0·6043	0·1794	0·0198	0·0072	40·1	..
120	0·5446	0·2391	0·0224	0·0098	40·9	...
180	0·5063	0·2774	0·0237	0·0111	40·0	...
Dark						
0	0·7837	...	0·0126	...	...	..
60	0·6468	0·1469	0·0136	0·0030	20·4	...
120	0·5656	0·2181	0·0173	0·0047	21·5	..
180	0·5345	0·2492	0·0178	0·0052	20·8	

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TABLE 14

100 gms of Ferric oxide + 0·8% as Wheat straw + 0·5% P$_2$O$_5$ as Tata basic slag

Period of Exposure in days	Total C Un-oxidised %	Total C Oxidised %	Total Nitrogen %	Increase in nitrogen %	Efficiency (amount of N fixed in mgm/gm of carbon oxidised)	Total available P$_2$O$_5$ %
Light						
0	0·7362	. . .	0·0118	. . .	. .	0·2314
60	0·5235	0·2127	0·0143	0·0109	51·2	0·2499
120	0·4604	0·2758	0·0261	0·0143	51·8	0·2594
180	0·4299	0·3063	0·0275	0·0157	51·2	0·2653
Dark						
0	0·7362		0·0118	. . .	. .	0·2314
60	0·5541	0·1821	0·0166	0·0048	26·3	0·2423
120	0·4917	0·2445	0 0185	0·0067	27·4	0·2498
180	0·4581	0·2781	0·0193	0·0075	26·9	0·2557

TABLE 15

100 gms of Ferric oxide + 0·8% C as Wheat straw + 0·5% P$_2$O$_5$ as Trichinopoly rock phosphate

Light						
0	0·7701	. .	0·0124	. .	. . .	0·0266
60	0·5653	0·2048	0·0219	0·0095	46·3	0·0381
120	0·5024	0·2677	0·0250	0·0126	47·0	0·0446
180	0·4672	0·3029	0·0264	0·0140	46·3	0·0484
Dark						
0	0·7701	. .	0·0124	. . .	. . .	0·0266
60	0·5484	0·1717	0·0165	0·0041	23·8	0·0347
120	0·5366	0·2335	0·0182	0·0058	24·8	0·0401
180	0·5018	0·2683	0·0189	0·0065	24·2	0·0433

TABLE 16

100 gms of Titania + 0·8% C as Wheat straw

Light						
0	0·7837	. . .	0·0126	. . .	. . .	. . .
60	0·5916	0·1921	0·0209	0·0083	43·2	. .
120	0·5309	0·2528	0·0257	0·0111	43·7	. .
180	0·4924	0·2913	0·0251	0·0125	42·9	. .
Dark						
0	0·7837	. . .	0·0126	. . .	. . .	. .
60	0·6251	0·1586	0·0161	0·0035	22·0	. . .
120	0·5526	0·2311	0·0180	0·0054	23·0	. . .
180	0·5228	0·2609	0·0185	0·0059	22·6	. . .

TABLE 17

100 gms of Titania + 0·8% C as Wheat straw + 0·5% P_2O_5 as Tata basic slag

Period of Exposure in days	Total C Un-oxidised %	Total C Oxidised %	Total Nitrogen %	Increase in nitrogen %	Efficiency (amount of N fixed in mgm/gm of carbon oxidised)	Total available P_2O_5 %
Light						
0	0·7362	..	0·0118	...	...	0·2314
60	0·5111	0·2551	0·0241	0·0123	54·6	0·2538
120	0·4477	0·2885	0·(.277	0·0159	55·1	0·2674
180	0·4172	0·3190	0·0292	0·0174	54·5	0·2756
Dark						
0	0·7362		0·0118	...	...	0·2314
60	0·5427	0·1935	0·0172	0·00ↄ4	27·9	0·2459
120	0·4799	0·2563	0·0192	0·0074	28·8	0·2573
180	0·4444	0·2918	0·0201	0·0083	28·4	0·2661

TABLE 18

100 gms of Titania + 0·8% C as Wheat straw + 0·5% P_2O_5 as Trichinopoly rock phosphate

Light						
0	0·7701	...	0·0124	...	...	0·0266
60	0·5520	0·2181	0·0233	0·0109	49·9	0·0412
120	0·4839	0·2812	0·0266	0·0142	50·5	0·0497
180	0·4537	0·3164	0·0282	0·0158	49·9	0·0551
Dark						
0	0·7701	...	0·0124	...	...	0·0266
60	0·5855	0·1846	0·0171	0·0047	25·4	0·0367
120	0·5240	0·2461	0·0189	0·0065	26·4	0·0441
180	0·4863	0·2838	0·0198	0·0074	26·0	0·0489

TABLE 19

100 gms of Zinc oxide + 0·8% C as Sawdust

Light						
0	0·7860	...	0·0084	...	...	...
60	0·6281	0·1579	0·0137	0·0053	33·5	...
120	0·5787	0·2073	0·0155	0·0071	34·2	...
180	0·5339	0·2521	0·0169	0·0085	33·3	..
Dark						
0	0·7860	...	0·0084	...	..	..
60	0·6582	0·1278	0·0106	0·0022	17·2	...
120	0·6016	0·1844	0·0118	0·0034	18·4	...
180	0·5662	0·2198	0·01ɔ3	0·0039	17·7	..

TABLE 20

100 gms of Zinc oxide + 0·8% C as Sawdust + 0·5% P_2O_5 as Tata basic slag

Period of Exposure in days	Total C Un-oxidised %	Total C Oxidised %	Total Nitrogen %	Increase in Nitrogen %	Efficiency (amount of N fixed in mgm/gm of carbon oxidised)	Total available P_2O_5 %
		Light				
0	0·7383	...	0·0079	...	..	0 2321
60	0·5488	0·1895	0·0164	0·0085	44·8	0·2497
120	0·4887	0·2496	0·0192	0·0113	45·2	0·2592
180	0·4502	0·2881	0·0208	0·0129	44·7	0·2655
		Dark				
0	0·7383	.	0·0079	...	...	0·2321
60	0·5832	0·1551	0·0115	0·0036	23·2	0·2426
120	0·5274	0·2109	0·0130	0·0051	24·1	0·2492
180	0·4922	0·2461	0·0137	0·0058	23·5	0·2543

TABLE 21

100 gms of Zinc oxide + 0·8% C as Sawdust + 0·5% P_2O_5 as Trichinopoly rock phosphate

		Light				
0	0·7724	...	0·0083	...	...	0·0266
60	0·5948	0·1778	0·0154	0·0071	39·9	0·0387
120	0·5343	0·2381	0·0179	0·0096	40·3	0·0449
180	0·4981	0·2743	0·0192	0·0109	39·7	0·0494
		Dark				
0	0·7724	...	0·0083	...	...	0·0266
60	0·6264	0·1460	0·0113	0·0030	20·4	0·0351
120	0·5735	0·1989	0·0125	0·0042	21·1	0·0404
180	0·5368	0·2356	0·0132	0·0049	20·7	0·0435

TABLE 22

100 gms of Ferric oxide + 0·8% C as Sawdust

		Light				
0	0·7860	...	0·0084	...		..
60	0·6366	0·1494	0·0131	0·0047	31·4	...
120	0·5872	0·1988	0·0148	0·0064	32·1	...
180	0 5421	0·2439	0·0160	0 0076	31·1	..
		Dark				
0	0·7860	...	0·0084		...	..
60	0·6666	0·1194	0·0104	0·0020	16·7	...
120	0·6102	0·1758	0·0115	0·0031	17·6	...
180	0·5756	0·2104	0·0119	0·0035	16·6	...

TABLE 23

100 gms of Ferric oxide + 0·8% C as Sawdust + 0·5% P_2O_5 as Tata basic slag

Period of Exposure in days	Total C Un-oxidised %	Total C Oxidised %	Total Nitrogen %	Increase in Nitrogen %	Efficiency (amount of N fixed in mgm/gm of carbon oxidised)	Total available P_2O_5 %
Light						
0	0·7383	. . .	0·0079	. . .	. . .	0·2321
60	0·5569	0·1814	0·0157	0·0078	42·9	0·2478
120	0·4968	0·2415	0·0184	0·0105	43·4	0·2556
180	0·4587	0·2796	0·0199	0·0120	42·9	0·2608
Dark						
0	0·7383	. . .	0·0079	. . .	. . .	0·2321
60	0·5918	0·1465	0·0112	0·0033	22·5	0·2408
120	0·5362	0·2021	0·0126	0·0047	23·2	0·2456
180	0·5011	0·2372	0·0133	0·0054	22·7	0·2495

TABLE 24

100 gms of Ferric oxide + 0·8% C as Sawdust + 0·5% P_2O_5 as Trichinopoly rock phosphate

Period of Exposure in days	Total C Un-oxidised %	Total C Oxidised %	Total Nitrogen %	Increase in Nitrogen %	Efficiency	Total available P_2O_5 %
Light						
0	0·7724	. . .	0·0083	. . .	. . .	0·0262
60	0·6033	0·1691	0·0147	0·0064	37·8	0·0365
120	0·5426	0·2298	0·0171	0·0088	38·2	0·0416
180	0·5063	0·2661	0·0183	0·0100	37·5	0·0449
Dark						
0	0·7724	. . .	0·0083	. . .	. . .	0·0266
60	0·6349	0·1375	0·0110	0·0027	19·6	0·0341
120	0·5820	0·1904	0·0122	0·0030	20·4	0·0386
180	0·5460	0·2264	0·0128	0·0045	19·8	0·0414

TABLE 25

100 gms of Titania + 0·8% C as Sawdust

Period of Exposure in days	Total C Un-oxidised %	Total C Oxidised %	Total Nitrogen %	Increase in Nitrogen %	Efficiency	Total available P_2O_5 %
Light						
0	0·7860	. . .	0·0084	. . .	. . .	. . .
60	0·6249	0·1611	0·0130	0·0056	34·7	. . .
120	0·5757	0·2103	0·0158	0·0074	35·1	. . .
180	0·5319	0·2541	0·0171	0·0087	34·2	. .
Dark						
0	0·7860	. . .	0·0084	. . .	. . .	. .
60	0·6548	0·1312	0·0108	0·0024	18·2	. . .
120	0·5992	0·1868	0·0120	0·0036	19·2	. . .
180	0·5632	0·2228	0·0125	0·0041	18·4	. .

TABLE 26

100 gms of Titania + 0·8% C as Sawdust + 0·5% P_2O_5 as Tata basic slag

Period of Exposure in days	Total C Un-oxidised %	Total C oxidised %	Total Nitrogen %	Increase in nitrogen %	Efficiency (amount of N fixed in mgm/gm of carbon oxidised)	Total available P_2O_5 %
Light						
0	0·7383	. . .	0·0079	. . .	. . .	0·2321
60	0·5452	0·1931	0·0168	0·0089	46·0	0·2502
120	0·4851	0·2532	0·0197	0·0118	46·6	0·2620
180	0·4465	0·2918	0·0213	0·0134	45·9	0·2683
Dark						
0	0·7383	. . .	0·0097	. . .	. . .	0·2321
60	0·5799	0·1584	0·0117	0·0038	23·9	0·2439
120	0·5241	0·2142	0·0132	0·0053	24·7	0·2518
180	0·4890	0·2493	0·0139	0·0060	24·0	0·2586

TABLE 27

100 gms of Titania + 0·8% C as Sawdust + 0·5% P_2O_5 Trichinopoly rock phosphate

Period of Exposure in days	Total C Un-oxidised %	Total C oxidised %	Total Nitrogen %	Increase in nitrogen %	Efficiency (amount of N fixed in mgm/gm of carbon oxidised)	Total available P_2O_5 %
Light						
0	0·7724	. . .	0·0083	. . .	. . .	0·0266
60	0·5912	0·1812	0·0157	0·0074	40·8	0·0398
120	0·5309	0·2415	0·0183	0·0100	41·4	0·0469
180	0·4943	0·2781	0·0197	0·0114	40·9	0·0520
Dark						
0	0·7724	. . .	0·0083	. . .	. . .	0·0266
60	0·6232	0·1492	0·0115	0·0032	21·4	0·0359
120	0·5703	0·2021	0·0128	0·0045	22·2	0·0419
180	0·5340	0·2384	0·0135	0·0052	21·8	0·0461

Discussion

From a close examination of the foregoing data it is seen that there is a decrease in the carbon content of the system with concomitant fixation of nitrogen when Glucose, Wheat straw and Sawdust are mixed with Zinc oxide, iron oxide and titania and allowed to undergo slow oxidation in air at ordinary temperature. It is observed that the oxidation of carbon and fixation of nitrogen are much greater in the sets exposed to light than in the sets kept in the dark, showing thereby the marked influence of light on the fixation of atmospheric nitrogen.

The rate of oxidation of carbon, fixation of nitrogen and efficiency, that is, the amount of nitrogen fixed in mgms per gram of carbon oxidised is in the order

Glucose > Wheat straw > Sawdust

As far as the surfaces are concerned, the activity is in the following order :

$$TiO_2 > ZnO > Fe_2O_3$$

This marked difference in the rate of carbon oxidation, nitrogen fixation and efficiency with the three surfaces can be due to the difference in their power of photo-sensitization. The photosensitizing action may be different in its nature from catalysts, as not only spontaneous reactions but also those involving an increase in the free energy of the system, can be realised by the introduction of suitable photosensitizers. The well known example of photo sensitization involving an increase in the free energy of the system is the carbon assimilation process sensitized by chlorophyll.

That nitrogen fixation is possible with chemically pure surfaces like Zinc oxide, ferric oxide and titania shows that soil is not absolutely necessary for nitrogen fixation. What really seems indispensable is a suitable surface, where water, oxygen, nitrogen and energy materials are property adsorbed and are in intimate contact in such a way that the oxidation of the energy materials is possible.

Moreover, it can be observed from the data, that when phosphates are added the rate of fall in the efficiency of nitrogen fixation is checked. It seems that phosphates play a vital role in the improvement and maintenance of the nitrogen status of soils. The proteins that are present in the soil humus or those formed due to the fixation of nitrogen are likely to be stabilised by the formation of more or less stable phospho-proteins with the combination of phosphates and proteins. It is also observed that phosphates enhance the oxidation of carbonaceous materials and a part of nitrogen may be fixed due to the release of this extra-energy.

The forgoing results, further, show that there is a considerable increase in the avilability of phosphate when phosphates are added to the system containing oxide surface and organic matter and allowed to undergo slow oxidation in air at ordinary temperature. It is also observed that the increase in the availability of phosphate varies with the surface, ferric oxide showing the least increase in comparison to Zinc oxide or titania. The process of phosphate fixation has received considerable attention and many theories have been advanced to explain it. The experimental results of Bass and Sieling[2], Bear and Toth[3], Cole and Jackson[4], Coleman[5], Ford[6], Haseman et al.[7] and many others [8,9,10] indicate that specific compounds of iron and Aluminium, which are relatively insoluble, are formed when soluble phosphates are added to acid soils. These compounds have been identified recently by Cole and Jackson[4] as Ferric dihydroxyl dihydrogen phosphate (Strengite,, and aluminium dihydroxyl dihydrogen phosphate (Variscite) or amorphous combinations of the two (Barrandite).

Many investigators [11,12,13,14,15] have observed that these retained phosphates are rendered soluble by the help of organic matter. The action of organic matter has been explained on the basis of the formation of fixation resisting organic phosphates[16], replacement of the phosphate caused by the humate part of organic matter[17] and the delay which it may cause in the absorption of phosphates by iron and aluminium[18].

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Complex Differential Systems and Conditional Stability

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1. It is a well-known result that Lyapunov's Second method depends essentially on the fact that a function satisfying a scalar differential inequality can be majorised by the maximal solution of the corresponding equation. This principle was extended by Wazewski [4] to vector differential inequalities, under certain monotonic restrictions. This comparison principle was used by Lakshmikantham[2] to derive sufficient conditions for conditional stability and boundedness of a set with respect to an ordinary differential system in the real domain.

Vejvoda[3] discussed the stability of differential equations in the complex domain, the independent variable being real. Kayande and Lakshmikantham[1], using the comparison principle stated above, obtained sufficient conditions for the stability and boundedness of the origin, with respect to a class of complex differential systems in which independent variable is also complex. We introduce in this paper the concepts of conditional stability and boundedness of a compact set with respect to a class of complex differential systems, and obtain sufficient conditions in terms of vector Lyapunov-like functions. This extends the work of Lakshmikantham[2] to complex systems and includes the results in[1] as special cases. Example is constructed to illustrate the results.

2. Let D denote the region of the complex plane : $0 \leqslant a \leqslant |z| < \infty$, $a \leqslant \arg z \leqslant \beta$, where a, α, β are real numbers. Let the interval $0 \leqslant a \leqslant t \leqslant \infty$ be denoted by I. Let C^n and R^n denote the complex and real n-space respectively. We shall say that a function $f \varepsilon (D, C^n)$ if it is defined on $D \times C^n$ into C^n and is regular-analytic on D and entire on C^n. Consider the complex differential system

$$(2 \cdot 1) \qquad y' = f(z, y) \; y(z_0) = y_0, z_0 \; \varepsilon \; D, \; \left(' \equiv \frac{d}{dz} \right)$$

where $f \varepsilon (D, C^n)$. By a solution of (2·1) we mean a complex n-vector function $y = y(z), y(z_0) = y_0, (z_0 \; \varepsilon \; D)$, that is regular-analytic in z on D and satisfies (2·1) for all $z \; \varepsilon \; D$. We assume the existence of solutions for (2·1).

3. Let a real n-vector function $w = w(t, r)$ be defined and continuous on $I \times R^n$. Let $r = (r_1, r_2, r_3, \ldots, r_n)$. The function w, defined above, is said to possess

property (P) if for each $t \in I$ and for each i, $(i = 1, 2, \ldots, n)$, the ith component of w denoted by $w_i(t, r)$, is non-decreasing in $r_1, r_2, \ldots, r_{i-1}, r_{i+1}, \ldots, r_n$. Let the function w possess the property (P). Then it is known[4] that the differential system

$$(3 \cdot 1) \qquad \dot{r} = w(t, r), \, r(t_0) = r_0, \, t_0 \in I, \left(\cdot \equiv \frac{d}{dt} \right)$$

has the maximal solution, in the sense of componentwise majorisation, existing to the right of t_0.

Let a function $V = V(z, y) \in (D, C^n)$. Define

$$(3 \cdot 2) \qquad V^*(z, y) = \frac{\partial V}{\partial z} + \frac{\partial V}{\partial y} \cdot f(z, y)$$

where $\cdot$ denotes scalar multiplication of vectors. For $y \in C^n$ let $|y|$ denote a vector in R^n such that $|y| = (|y_1|, |y_2|, \ldots, |y_n|)$ $|y_i|$ is the modulus of y_i, the ith component of y. The following lemma is established in[1].

Lemma : Let there exist a function $V = V(z, y) \in (D, C^n)$ such that $V^*(z, y)$ of $(3 \cdot 2)$ satisfies the vector inequality

$$|V^*(z, y)| \leqslant w(|z|, |V(z, y)|)$$

where w has the monotonic property (P). Suppose $r(t)$ is the maximal solution of the system $(3 \cdot 1)$. If $y(z)$ is any solution of $(2 \cdot 1)$ with

$$|V(z_0, y(z_0))| \leqslant r_0, \, (|z_0| = t_0),$$

then

$$|V(z, y(z))| \leqslant r(t), \, (z \in D, |z| = t), \, t \geqslant t_0.$$

Note : Any two vectors belonging to R^n are said to satisfy vector inequality if the same inequality holds for each of the corresponding components of the vectors.

4. In this section we formulate in a natural way conditions for conditional stability and boundedness of solutions of $(2 \cdot 1)$ with respect to a compact set $A \subset C^n$. We assume that A contains the origin. Define

$$d(y, \bar{y}) = \sum_{i=1}^{n} |y_i - \bar{y}_i|, \text{ for } y, \bar{y} \in C^n. \text{ Let}$$

$d(y, A) = \inf_{\bar{y} \in A} d(y, \bar{y})$. The sets $\{ y : d(y, A) < a \}$ and

$\{ y : d(y, A) \leqslant a \}$ will be denoted by $S(A, a)$ and $\overline{S(A, a)}$

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respectively. Let M be a set such that $A \subset M \subset C^n$. A is said to be invariant for system (2·1), if $y(z_0) \varepsilon A \Longrightarrow y(z) \varepsilon A$, for all $z \varepsilon D$. We state the following conditions.

(i) For any arbitrary $\varepsilon > 0$, $z_0 \varepsilon D$, there exist a positive function $\delta = \delta(|z_0|, \varepsilon)$ that is continuous in $|z_0|$ for each ε such that

$$y(z) \subseteq S(A, \varepsilon) \quad (z \varepsilon D, |z| \geqslant |z_0|)$$

whenever

$$y(z_0) \varepsilon \overline{S(A, \delta)} \cap M.$$

(ii) The δ in (i) is independent of $|z_0|$, $z_0 \varepsilon D$.

(iii) For any arbitrary $\varepsilon > 0$, $\gamma \geqslant 0$ and $z_0 \varepsilon D$, there exist a positive number $T = T(|z_0|, \gamma, \varepsilon)$ such that

$$y(z) \subset S(A, \varepsilon) \ (z \varepsilon D, |z| \geqslant |z_0| + T)$$

whenever

$$y(z_0) \varepsilon \overline{S(A, \gamma)} \cap M.$$

(iv) The T in (iii) is independent of $|z_0|$.

(v) Conditions (i) and (iii) hold simultaneously.

(vi) Conditions (ii) and (iv) hold simultaneously.

(vii) For each $\gamma \geqslant 0$, $z_0 \varepsilon D$, there exists a positive function $\eta = \eta(|z_0|, \gamma)$ that is continuous in $|z_0|$ for each γ, such that

$$y(z) \subseteq S(A, \eta), \ (z \varepsilon D, |z| \geqslant |z_0|)$$

whenever

$$y(z_0) \varepsilon \overline{S(A, \gamma)} \cap M.$$

(viii) The η in (vii) is independent of $|z_0|$.

(ix) For each $\gamma > 0$ and $z_0 \varepsilon D$ there exist positive numbers B and $T = T(|z_0|, \gamma)$ such that

$$y(z) \subseteq S(A, B), \ (z \varepsilon D, |z| \geqslant |z_0| + T)$$

whenever

$$y(z_0) \varepsilon \overline{S(A, \gamma)} \cap M.$$

(x) The T in (ix) is independent of $|z_0|$.

(xi) The conditions (vii) and (ix) hold simultaneously.

(xii) The conditions (viii) and (x) hold simultaneously.

Note: These conditions are natural extensions of the corresponding conditions of conditional stability and boundedness for real ordinary systems with respect to a set A[2]. If $M = C^n$ the conditions reduce to the corresponding conditions of stability and boundedness for complex systems with respect to

a set A. However, if $M = C^n$ and $A = \{0\}$, these reduce to the corresponding conditions in[1]. We are not assuming that A is invariant nor uniqueness of solutions. But the invariancy or otherwise of A is implied by the conditions. For example, if the condition (i) holds and $M = C^n$ then A must be invariant.

5. Let $\| V(z, y) \| = \sum\limits_{i=1}^{n} | V_i(z, y) |$. The following assumptions will be used

subsequently whenever necessary.

(5·1) For $t_0 \varepsilon I$, $r(t_0) = r_0 = (r_{10}, r_{20}, \ldots, r_{k0}, 0, 0, \ldots, 0)$, $r_0 \varepsilon R^n$, $k \leqslant n$.

(5·2) There exist a scalar function $p(z)$ that is regular-analytic in z on D such that a function $V \varepsilon (D, C^n)$ satisfying the conditions of the lemma equals $p(z) V_1(z, y)$, where $V_1 \varepsilon (D, C^n)$. For fixed

$$|z|, \quad \min_{\alpha \leqslant \arg z \leqslant \beta} |p(z)| = \phi(t), |z| = t, \text{ and } \phi(t) \to \infty \text{ as } t \to \infty.$$

Also V_1 satisfies (5·5).

(5·3) There exists a continuous and non-decreasing function $b = b(r)$, $r \geqslant 0$, $b(r) > 0$ for $r > 0$ and $b(d(y, A)) \leqslant \| V(z, y) \|$ for all $(z, y) \varepsilon D \times C^n$, and $b(r) \to \infty$ as $r \to \infty$.

(5·4) $\| V(z, y) \| \to 0$ as $d(y, A) \to 0$ for each $z \varepsilon D$.

(5·5) $\| V(z, y) \| \to 0$ as $d(y, A) \to 0$ uniformly in $z \varepsilon D$.

(5·6) $V(z, y) \equiv 0$ whenever $y \varepsilon A$.

(5·7) $M = \{ y : V_i(z, y) \equiv 0, i = k + 1, \ldots, n \}$.

Corresponding to the conditions (i), (ii), (vii) and (viii) given in section 4, following conditions are stated for the system (3·1).

(ia) Given $\varepsilon > 0$ and $t_0 \varepsilon I$, there exists a positive function $d = d(t_0, \varepsilon)$ that is continuous in t_0, for each ε, such that

$$\sum_{i=1}^{n} r_i(t) < \varepsilon \ (t \geqslant t_0),$$

whenever

$$\sum_{i=1}^{n} r_{i0} \leqslant d, \text{ where } r_0 \text{ satisfies (5·1)}.$$

Conditions (iia), (viia) and (viiia) can be similarly formulated. For conditions (iiia), (iva), (ixa) and (xa) it is assumed that (5·2) holds. We state the condition (iiia) only. Others can be similarly formulated.

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(iiia)   Given $\varepsilon > 0$, $\alpha \geqslant 0$ and $t_0 \in I$, there exists a positive number $T = T\,(t_0,\varepsilon,\alpha)$ such that

$$\sum_{i=1}^{n} r(t) < \varepsilon\,\phi(t),\ t \geqslant t_0 + T,$$

whenever

$$\sum_{i=1}^{n} r_{10} \leqslant \alpha\,\phi(t_0)$$

where r_0 satisfies (5·1) and $\phi(t)$ is as defined in (5·2).

We state and prove the following theorem :

Theorem 1 :   Let the assumptions of the lemma hold together with (5·3), (5·4), (5·6) and (5·7), then

(I) condition *(ia)* $\Longrightarrow$ condition *(i)*,

(II) condition *(viia)* $\Longrightarrow$ condition *(vii)*.

If the condition (5·4) is strengthened to (5·5), then

(III) condition *(iia)* $\Longrightarrow$ condition *(ii)*,

(IV) condition *(viiia)* $\Longrightarrow$ condition *(viii)*.

Proof :   Consider   *(I)*.   For any given $\varepsilon > 0$, if $d(y, A) = \varepsilon$, (5·3) shows that

(5·8) $$b(\varepsilon) \leqslant \|\,V(z,y)\,\|\,.$$

If *(ia)* holds, given $b(\varepsilon) > 0$ and $t_0 \in I$, there exists a positive function $d = d(t_0, \varepsilon)$ that is continuous in t_0 for each ε, such that

(5·10) $$\sum_{i=1}^{n} r_i(t) < b(\varepsilon),\ t \geqslant t_0$$

whenever

$$\sum_{i=1}^{n} r_{i0} \leqslant d,\ \text{where}\ r_0\ \text{satisfies (5·1)}.$$

In view of (5·4) $\exists$ a $\delta = \delta(\,|z_0|\,, d) = \delta(\,|z_0|\,, \varepsilon)$ such that

(5·11) $$\sup_{d(y_0,\,A)\,\leqslant\,\delta} \|\,V(z_0, y(z_0)\,)\,\| \leqslant d$$

where $$y_0 = y(z_0).$$

Due to vector inequality it follows from the lemma that

(5·12) $$\|\,V(z, y(z))\,\| \leqslant \sum_{i=1}^{n} r_i(t),\ (\,|z| = t \geqslant t_0)$$

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whenever

(5·13) $$|V_i(z_0, y_0)| \leqslant r_{i_0}, \quad (i = 1, 2, \ldots, n).$$

But (5·13) together with (5·1) and (5·7) imply that

$$y_0 \in M.$$

We claim that whenever $y_0 \in \overline{S(A, \delta)} \cap M$, every solution $y(z)$ of (2·1) satisfies $y(z) \subset S(A, \epsilon)$ i.e. $d(y(z), A) < \epsilon$ for all $z \in D, |z| \geqslant |z_0|$. Otherwise there exists a solution $y(z)$ with $y_0 \in \overline{S(A, \delta)} \cap M$ such that for some $z_1 \in D, |z_1| \geqslant |z_0|$, $d(y(z_1), A) = \epsilon$. But this fact together with (5·11), (5·13), (5·12) and (5·10) leads to the contradiction

$$b(\epsilon) \leqslant \| V(z_1, y(z_1)) \| \leqslant \sum_{i=1}^{n} r_i(t_1) < b(\epsilon).$$

Consider now proposition (II). Let $\gamma \geqslant 0$, $z_0 \in D$ be given
Due to (5·1) $\exists$ a $\gamma_1 = \gamma_1(|z_0|, \gamma)$ satisfying

(5·14) $$\sup_{d(y_0, A) \leqslant \gamma} \| V(z_0, y_0) \| \leqslant \gamma_1.$$

As condition (viia) holds, given $\gamma_1 \geqslant 0$ and $t_0 \in I$, there is a positive function $\eta = \eta(t_0, \gamma_1) = \eta(t_0, \gamma)$ that is continuous in t_0 for each γ, with property that

(5·15) $$\sum_{i=1}^{n} r_i(t) < \eta \text{ for all } t \geqslant t_0$$

whenever

(5·16) $$\sum_{i=1}^{n} r_{i_0} \leqslant \gamma.$$

where r_0 satisfies (5·1). Let $y_0 \in \overline{S(A, \gamma)} \cap M$. Then due to (5·14), (5·1) and (5·16) every solution $y(z)$ of (2·1), with $y(z_0) = y_0$, satisfies (5·12) by a proper choice of r_0. As $b(r) \to \infty$ as $r \to \infty$ $\exists$ a $L = L(\eta)$ such that

(5·17) $$\eta \leqslant b(L).$$

Then every solution $y(z)$ of (2·1), with $y_0 \in \overline{S(A, \gamma)} \cap M$, satisfies $d(y(z), A) < L$ for all $z \in D, |z| \geqslant |z_0|$. For otherwise $\exists$ a solution $y(z)$ of (2·1), with $y_0 \in \overline{S(A, \gamma)} \cap M$, and a $z_1 \in D, |z_1| \geqslant |z_0|$, such that $d(y(z_1), A) = L$. But this leads to the contradiction

$$b(L) \leqslant \| V(z_1, y(z_1)) \| \leqslant \sum_{i=1}^{n} r_i(t_1) < \eta \leqslant b(L),$$

due to (5·3), (5·12), (5·15) and (5·17).

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Proofs for propositions III and IV follow easily from the one given above, for due to the uniformity conditions (iia), $(viiia)$ and $(5\cdot5)$, the δ and L are independent of t_0. We state the following theorem without proof as the arguments follow similar to those in [1], with the necessary modifications indicated in Theorem 1.

Theorem 2: Let the assumptions of the lemma hold together with $(5\cdot2)$, $(5\cdot3)$, $(5\cdot4)$, $(5\cdot6)$ and $(5\cdot7)$. Then

(V) condition $(iiia) \Longrightarrow$ condition (iii)

(VI) condition $(ixa) \Longrightarrow$ condition (ix).

If the assumption $(5\cdot4)$ is strengthened to $(5\cdot5)$, then

(VII) condition $(iva) \Longrightarrow$ condition (iv).

(VIII) condition $(xa) \Longrightarrow$ condition (x).

Combining the results of Theorems 1 and 2 we show that the other conditions listed in section 4 also have these properties.

6. In this section we give an example to illustrate the results

Example : Consider the system

$$y_1' = \alpha y_1 + \beta y_2$$

(6·1)

$$y_2' = \gamma y_1 + \delta y_2,$$

where α, β, γ, δ are regular-analytic functions of z on a certain domain D of the complex plane. Let $V_1 = p(z)\,(y_1 - y_2)$ and $V_2 = p(z)\,(y_1 + y_2)$, where $p(z)$ is regular-analytic in z on D and satisfies the condition of $(5\cdot2)$, with $|p(z)| \geqslant 1$ on D. Now $\|V\| = |p(z)|\,\{\,|y_1 + y_2|\,\} \geqslant |y_1 + y_2| + |y_1 - y_2| \geqslant |y_1| + |y_2| \geqslant \|y\|$. Hence the function b of $(5\cdot3)$ can be the identity function *i.e.* $b(r) = r$. Also $(5\cdot4)$ holds. We have from $(3\cdot2)$

$$V_1^* = \alpha_1 p(z)\,(y_1 + y_2) + [\beta_1 p(z) + p'(z)]\,[y_1 - y_2],$$
$$V_2^* = [(\alpha + \gamma)\,y_1 + (\beta + \delta)\,y_2]\,p(z) + p'(z)\,(y_1 + y_2)$$

where

$$\alpha_1 = \frac{\alpha + \beta - \gamma - \delta}{2} \text{ and } \beta_1 = \frac{\alpha - \beta - \gamma + \delta}{2}.$$

Assume that α, β, γ, δ are so chosen that $|\alpha_1\,p(z)| \leqslant \psi(|z|)$,

$|\beta_1\,p(z) + p'(z)| \leqslant \phi(|z|)$ and $|V_2^*| \leqslant \theta(|z|)\,|y_1 + y_2|$. Then the system $(3\cdot1)$ reduces to

$$\dot{r}_1 = \phi(t)\,r_1 + \psi(t)\,r_2$$

(6·2)

$$\dot{r}_2 = \theta(t)\,r_2.$$

The unique solution of $(6\cdot2)$ through $r_0 = (r_{10}, r_{20})$ is given by

$$r_1(t) = r_{10} \exp \int_{t_0}^{t} \phi(s)\,ds + r_{20} \int_{t_0}^{t} \psi(s) \exp \left[\int_{t_0}^{t} \theta(u)\,du \right.$$

$(6\cdot3)$ $\qquad + \int_s^t \phi(u)\, du \bigg]\, ds, \quad r_2(t) = r_{20} \exp \int_{t_0}^t \theta(s)\, ds.$

Let

$$\exp \int_L^t \theta(s)\, ds \to \infty \text{ as } t \to \infty.$$

Then taking $k = 1$, $r_0 = (r_{10}, 0)$ the system $(6\cdot3)$ reduces to

$$r_1(t) = r_{10} \exp \int_{t_0}^t \phi(s)\, ds,$$

$(6\cdot4)$ $\qquad\qquad\qquad r_2(t) \equiv 0.$

Also from $(5\cdot7)$ the set M is the set of points $\{ y : y_1 + y_2 = 0 \}$, A being the set $\{ 0 \}$. Obviously $A \subset M$. All the required assumptions of the theorems are satisfied.

Illustration 1 : Let $\displaystyle\int_{t_0}^t \phi(s)\, ds \leqslant M(t_0) < \infty$, then $(6\cdot2)$ has the property (ia).

Theorem 1 shows that the system $(6\cdot1)$ has the property (i), with A and M as defined above.

Illustration 2 : Let $\displaystyle\min_{z \,\varepsilon\, D,\, \arg z} |p(z)| = P(t)$, $t = |z|$.

Then

$$\frac{\exp \displaystyle\int_{t_0}^t \phi(s)\, ds}{P(t)} \to 0 \text{ as } t \to \infty$$

implies that $(6\cdot2)$ has the property $(iiia)$, Then the application of Theorem 2 shows that $(6\cdot1)$ has the property (iii).

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A relation between generalised Kontorovitch-Lebdev transform and Weyl (Fractional) integral

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Abstract

The aim of this paper is to establish a relation between Generalised Kontorovitch Lebdev Transform and Weyl (Fractional) Integral. Certain new Weyl (Fractional) Integrals are solved with the help of this relation.

1. Introduction

The Generalised Kontorovitch-Lebdev Transform defined by Jet Wimp in his paper, "A Class Of Integral Transform," [(1) ; p. 37 ; (4·9) and (4·10)], is defined as follows :

$$f(x) \equiv T_{a:x}^{k} : g(t) = \left(\frac{\pi}{ax}\right)^{1/2} \int_{0}^{\infty} W_{k, it}(ax)\, g(t)\, dt \qquad (1·1)$$

and its inversion formula :

$$g(t) = \frac{a}{\pi^{5/2}}\, t \sinh(2\pi t)\, \Gamma(\tfrac{1}{2} - k + it)\, \Gamma(\tfrac{1}{2} - k - it)$$

$$\times \int_{0}^{\infty} (ax)^{-3/2}\, W_{k,\, it}(ax)\, f(x)\, dx \qquad (1·2)$$

The aim of the present paper is to establish a relation between the above transform and the Weyl (Fractional) Integral, defined as :

$$K_{y}^{\mu} : \{f(x)\} = [\,\Gamma(\mu)\,]^{-1} \int_{0}^{\infty} f(x)\, (x - y)^{\mu-1}\, dx \qquad (1·3)$$

Further, this relation between (1·1) and (1·3) is used to get the solution of certain Weyl (Fractional) Integrals, which are supposed to be new.

By virtue of the identity :

$$W_{0, \mu}(x) = \left(\frac{\pi}{x}\right)^{\frac{1}{2}} K_{\mu}\left(\frac{x}{2}\right) \qquad (1·4)$$

(1·1) reduces to the Kontorovitch-Lebdev Transform :

$$f(x) \equiv T_{2:x}^{0} : \{g(t)\} = \int_{0}^{\infty} K_{it}(x)\, g(t)\, dt \qquad (1·5)$$

when we put $k = 0$ and $a = 2$.

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2. Theorem

If
$$R(\mu) > 0, \quad R(a) > 0 \quad \text{and} \quad g(t) \; \varepsilon \; L(0, \infty)$$
then

$$T \, \substack{k-\mu \\ a:y} : \{ g(t) \} = e^{1/2 \, ay}. \, y^{1/2-k} \, K \, \substack{\mu \\ y} : [\, e^{-\frac{1}{2} ax}. \, x^{k-\mu-\frac{1}{2}}. \, T \, \substack{k \\ a:x} : \{ g(t) \}] \qquad (2 \cdot 1)$$

Proof :

Since, by hypothesis :

$$T \, \substack{k \\ a:x} : \{ g(t) \} = \left(\frac{\pi}{ax} \right)^{\frac{1}{2}} \int_0^\infty W_{k,it}(ax) \, g(t) \, dt \qquad (2 \cdot 2)$$

Multiplying both the sides by $\{ e^{-\frac{1}{2} ax}. \, x^{k-\mu-\frac{1}{2}}. \, (x - y)^{\mu-1} \}$ and integrating with respect to x within the limits y to ∞, we have :

$$\int_y^\infty e^{-\frac{1}{2} ax}. \, x^{k-\mu-\frac{1}{2}} \, (x-y)^{\mu-1} \, T \, \substack{x \\ a:x} : \{ g(t) \} \, dx$$

$$= \left(\frac{\pi}{a} \right)^{\frac{1}{2}} \int_y^\infty e^{-\frac{1}{2} ax}. \, x^{k-\mu-1}. \, (x-y)^{\mu-1}. \int_0^\infty W_{k,it} (ax) \, g(t) \, dt. \, dx. \qquad (2 \cdot 3)$$

Changing the order of integration, we have :

$$\int_y^\infty e^{-\frac{1}{2} ax}. \, x^{k-\mu-\frac{1}{2}}. \, (x-y)^{\mu-1}. \, T \, \substack{k \\ ax} : \{ g(t) \} \, dx$$

$$= \left(\frac{\pi}{a} \right)^{\frac{1}{2}} \int_0^\infty g(t) \, dt \int_y^\infty e^{-\frac{1}{2} ax}. \, x^{k-\mu-1}. \, W_{k,it}(ax) \, (x-y)^{\mu-1}. \, dx \qquad (2 \cdot 4)$$

Solving the inner integral on the right hand side with the help of the result [(2) ; p. 211 ; (73)] ; *viz* :

$$K \, \substack{\mu \\ y} : \{ x^{k-\mu-1}. \, e^{-\frac{1}{2} ax}. \, W_{k,\lambda} \, (ax) \} = y^{k-1}. \, e^{-\frac{1}{2} ay}. \, W_{k-\mu,\lambda}(ay) \qquad (2 \cdot 5)$$

where
$$R(\mu) > 0, \quad R(ay) > 0,$$
we have :

$$[\Gamma(\mu)]^{-1} \int_y^\infty \{ e^{-\frac{1}{2} ax}. \, x^{k-\mu-\frac{1}{2}}. \, T \, \substack{k \\ ax} : g(t) \} \, (x-y)^{\mu-1}. \, dx$$

$$= e^{-\frac{1}{2} ay}. \, y^{k-\frac{1}{2}}. \, T \, \substack{k-\mu \\ ay} : \{ g(t) \}. \qquad (2 \cdot 6)$$

which is the required result.

The proof of the theorem involves the change of order of integration. To justify the same, we observe that since by [(3) ; p. 264 ; (5)] and [(3); p. 183; (1)]:

$$W_{k,it}(ax) = e^{-ax/2}. \, (ax)^k \, {}_2F_0 \left(\tfrac{1}{2} - k + it, \, \tfrac{1}{2} - k - it \, ; \, -\frac{1}{ax} \right)$$

$$= e^{-ax/2}. \, (ax)^k \sum_{n=0}^\infty \frac{\Gamma(\tfrac{1}{2} - k + it + n) \, \Gamma(\tfrac{1}{2} - k - it + n)}{(n) ! \, \Gamma(\tfrac{1}{2} - k + it) \, \Gamma(\tfrac{1}{2} - k - it)} \left(-\frac{1}{ax} \right)^n$$

If we use [(3) ; p. 47 ; (6)] ; *viz.* :

$$\underset{|y| \to \infty}{Lim} \left| \Gamma(x+iy) \right| = (2\pi)^{\frac{1}{2}} . e^{\frac{1}{2}\pi |y|} . |y|^{\frac{1}{2}-x} ; x, y \text{ real}$$

we have :

$$\underset{t \to \infty}{Lt} W_{k, it}(ax) = \underset{t \to \infty}{Lt} e^{-ax/2} . (ax)^{k} . e^{-[ax|t^2|]^{-1}}$$

Hence, $W_{k, it}(ax)$ is bounded for large values of t, and for finite a and x.

Therefore, the t – integral in the equation (2·3) is absolutely convergent if $g'(t) \, \varepsilon \, L(0, \infty)$ as $W_{k, it}(ax)$ shall be bounded for large values of t ; and the integrand in the x–integral, in the same equation (2·3), for large values of x is comparable $|e^{-ax} . x^{2(k-1)}|$ since :

$$W_{k, m}(x) \sim A e^{-\frac{1}{2}x} . x^{k} ; \text{ for large values of } x.$$

Hence, the x–integral will be convergent if $R(a) > 0$.

The resulting integral on the left of (2·3) will be convergent if $g(t) \, \varepsilon \, L(0, \infty)$ by the same argument written before.

3. Examples

(*i*) Let us start with

$$g(t) = t \sinh(2\pi t) \Gamma(\tfrac{1}{2} - k + it) \Gamma(\tfrac{1}{2} - k - it) \Gamma(k + \rho + it - \tfrac{1}{2}) \Gamma(k + \rho - it - \tfrac{1}{2})$$
$$\times W_{1-k-\rho, it}(ay), \tag{3·1}$$

then from the inversion formula, we have :

$$f(x) \equiv T_{ax}^{k} : \{g(t)\} = \pi^{5/2} . a^{\frac{1}{2}} . z^{1-k} . \Gamma(\rho) . e^{-\frac{a}{2}(x+z)} . x^{k+\rho-\frac{1}{2}} . (x+z)^{-\rho} \tag{3·2}$$

where $|\arg z| < \pi$, $R(a) > 0$, $R(\rho) > 0$; $R(k + \rho - 2) > -3/2$

Hence, (2·1) yields :

$$[\Gamma(\mu)]^{-1} \int_{y}^{\infty} \{e^{-ax} . x^{2k+\rho-1-\mu} . (x+z)^{-\rho}\} (x-y)^{\mu-1} . dx$$
$$= e^{-ay} . y^{2k+\rho-1-\mu} . (y+z)^{-\rho} . z^{-\mu}. \tag{3·3}$$

where : $|\arg z| < \pi$, $R(a) > 0$, $R(\rho) > 0$, $R(\mu) > 0$ and $R(k+\rho-2) > -3/2$.

(*ii*) Let us start with :

$$g(t) = t \sinh(2\pi t) \Gamma(\tfrac{1}{2} - k + it) \Gamma(\tfrac{1}{2} - k - it) \Gamma(\tfrac{1}{2} + \nu + it + \rho)$$

$$\times \Gamma(\tfrac{1}{2} + \nu - it + \rho) \, {}_3F_2 \left(\begin{array}{c} \tfrac{1}{2} + \lambda + \nu, \ \tfrac{1}{2} + \nu + it + \rho, \ \tfrac{1}{2} + \nu - it + \rho \\ 2\nu + 1, \ 1 - k + \nu + \rho \end{array} ; -\frac{2b}{a} \right) \tag{3·4}$$

then from the inversion formula, we have :

$$f(x) \equiv T_{ax}^{k} : \{g(t)\} = \pi^{5/2} . (2b)^{-\nu - \frac{1}{2}} . a^{\nu + \rho} . \Gamma(1 - k + \nu + \rho) . (ax)^{\rho}$$

$$\times \exp. [-\tfrac{1}{2}(a+2b)x] M_{\lambda, \nu}(2bx) \tag{3·5}$$

$$R(b) > 0, \ R(a) > 0, \ R(\rho - \tfrac{1}{2} + \nu) > -1.$$

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Hence, from (2·1) we have :

$$[\Gamma(\mu)]^{-1} \int_y^\infty \{ e^{-(a+b)x} . x^{k+\rho-\mu-\frac{1}{2}} . M_{\lambda,\nu}(2bx) \} (x-y)^{\mu-1} . dx$$

$$= \frac{\Gamma(1-k-\mu+\nu+\rho)}{\Gamma(1-k+\nu+\rho)} y^{k+\rho-\frac{1}{2}} . \exp. [-(a+b)y] . M_{\lambda,\nu}(2by) \qquad (3\cdot6)$$

where $\qquad R(b) > 0, \; R(a) > 0, \; R(\rho - \frac{1}{2}+\nu) > -1, \; R(\mu) > 0$

(iii) Let us start with

$$g(t) = t \sinh(2\pi t) \Gamma(\tfrac{1}{2} - k + it) \Gamma(\tfrac{1}{2} - k - it)$$

$$\times \; G_{33}^{23} \left(2 \; \middle| \; \begin{array}{c} \frac{1}{2}+it, \; \frac{1}{2}-it, \; \frac{1}{2}+k+\rho \\ \nu+\rho, \; -\nu+\rho, \; k \end{array} \right) \qquad (3\cdot7)$$

then from the inversion formula, we have :

$$f(x) \equiv T_{ax}^{k} : \{g(t)\} = \pi^{5/2} . 2^{\rho-\frac{1}{2}} . a^{2\rho-\frac{1}{2}} . \Gamma(\tfrac{1}{2} - k + \nu)$$

$$\times \; \Gamma(\tfrac{1}{2} - k - \nu) . x^{\rho} . e^{ax/2} . W_{k,\nu}(2ax). \qquad (3\cdot8)$$

$$R(a) > 0, \; R(\mu) < R(\rho - \tfrac{1}{2}).$$

Hence, substituting to values of $T_{ax}^{k} : \{g(t)\}$ and $T_{ay}^{k-\mu} : \{g(t)\}$ in (2·1), we have :

$$[\Gamma(\mu)]^{-1} \int_y^\infty \{ x^{\rho+k-\mu-\frac{1}{2}} . W_{k,\nu}(2ax) \} (x-y)^{\mu-1} . dx$$

$$= \frac{\Gamma(\tfrac{1}{2} - k - \mu + \nu) \; \Gamma(\tfrac{1}{2} - k - \mu - \nu)}{\Gamma(\tfrac{1}{2} - k + \nu) \; \Gamma(\tfrac{1}{2} - k - \nu)} y^{\rho+k-\frac{1}{2}} . W_{k-\mu,\nu}(2ay) \qquad (3\cdot9)$$

with the conditions :

$$R(a) > 0, \quad 0 < R(\mu) < R(\rho+\tfrac{1}{2}).$$

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An Inversion Formula for H-function Transform

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Abstract

The aim of the present paper is to generalise the results given by Jet Wimp in his paper, "A Class Of Integral Transform," *Proc. Edin. Math. Soc*, (1), 14 (19.4) and to derive an inversion formula for H-function.

1. Introduction

Recently Jet Wimp has generalised the Kontorovitch Lebdev transform pair, [(2) ; p. 173], and the generalised Mehler transform pair [(5)]. These transforms are used in solving certain boundary value problems. The obj ct of this paper is to generalise futher the transform pair [(3) ; p. 36 ; (3·9) and (2·10)], given by Jet Wimp, whose kernel involves a function defined by Charles Fox [4) ; p. 408], written below, slightly in a different form, *viz* :

$$H^{m,\,n}_{p,\,q}\left[x \left| \begin{array}{l} (a_1, e_1)\,(a_2, e_2),\,\ldots\ldots\ldots, (a_p, e_p) \\ (b_1, f_1),\,(b_2, f_2);\,\ldots\ldots\ldots, (b_q, f_q), \end{array} \right. \right]$$

$$= \frac{1}{2\pi i} \int_L \frac{\overset{m}{\underset{j=1}{\pi}} \Gamma(b_j - f_j s) \overset{n}{\underset{j=1}{\pi}} \Gamma(1 - a_j + e_j s)}{\overset{q}{\underset{j=1}{\pi}} \Gamma(1 - b_j + f_j s) \overset{p}{\underset{j=1}{\pi}} \Gamma(a_j - e_j s)} x^s\, ds \qquad (1\cdot1)$$

where p, q, m, n are all non negative integers: e's and f's are all positive and an empty product is interpreted as 1 ; L is a suitable contour of Barnes type such that the poles of $\Gamma(b_j - f_j\xi)\, j = 1, \ldots\ldots, m$ lie on the right hand side of the contour and those of $\Gamma(1 - a_j + e_j\xi)\, ; j = 1, \ldots\ldots, n$ lie on the left hand side. Also the parameters are so restricted that the integral on the right of (1·1) is convergent.

Symbolically, we will write (1·1) as :

$$H^{m,\,n}_{p,\,q}\left[x \left| \begin{array}{l} [\,a_p, e_p\,] \\ [\,b_q, f_q\,] \end{array} \right. \right] \qquad (1\cdot2)$$

where $[\,a_p, e_p$ stands for an ordered set of parameters

$$(a_1, e_1), (a_2, e_2), \ldots\ldots, (a_p, e_p). \qquad (1\cdot3)$$

Section 2, contains some preliminary results and definitions and Section 3, contains the main result and its derivation.

2. Some definitions and results

Throughout this paper, we shall denote by $M_s\{f(x)\}$ and $M_x^{-1}\{g(s)\}$ the Mellin transform of $f(x)$ and the inverse Mellin Transform of $g(s)$, respectively, i.e.

$$g(s) = M_s\{f(x)\} = \int_0^\infty x^{s-1} f(x)\, dx \tag{2.1}$$

and

$$f(x) = M_x^{-1}\{g(s)\} = \frac{1}{2\pi i} \int_{c+i\infty}^{c+i\infty} x^{-s} g(s)\, ds. \tag{2.2}$$

In what follows, λ, μ and ν are given by :

$$\lambda = \frac{b-a}{2}, \ \mu = \frac{b-1}{A} \tag{2.3}$$

and

$$\nu = 2A + \sum_{j=1}^{m} (f_j) - \sum_{j=1}^{q} (\sigma_j) + \sum_{j=1}^{n} (e_j) - \sum_{j=1}^{p} (\varepsilon_j) \tag{2.4}$$

where A is a positive integer.

By virtue of the results 1.1) and (2.2), we have

$$M(s) = M_s\left\{ H_{n+p+2,\, m+q}^{m,\, n+2}\left[x \ \Big| \ {\begin{matrix} (1-\lambda, A),\ (\tfrac{1}{2}-\lambda, A),\ [a_n-\mu e_n,\ e_n],\ [\alpha_p-\mu\varepsilon_p,\ \varepsilon_p] \\ [b_m-\mu f_m,\ f_m],\ [\beta_q-\mu\sigma_q,\ \sigma_q] \end{matrix}} \right] \right\}$$

$$= \frac{\prod\limits_{j=1}^{m} \Gamma(b_j - \mu f_j + f_j s) \prod\limits_{j=1}^{n} \Gamma(1 - a_j + \mu e_j - e_j s)\, \Gamma(\lambda - As)\, \Gamma(\tfrac{1}{2} + \lambda - As)}{\prod\limits_{j=1}^{q} \Gamma(1 - \beta_j + \mu\sigma_j - \sigma_j s) \prod\limits_{j=1}^{p} \Gamma(\alpha_j - \mu\varepsilon_j + \varepsilon_j s)} \tag{2.5}$$

where

(1) m, n, p and q are all non negative integers,

(2) $\nu > 0$; value of ν is given by (2.4),

(3) max. $R\left[\dfrac{a_k - 1}{e_k}\ ;\ \lambda\right] < R(\mu - \lambda) < \min R\left[\dfrac{b_h}{f_h}\right]$

$\qquad k = 1, 2, \ldots, n$ and $h = 1, 2, \ldots, m$.

Again, by virtue of the result (2.2), we have :

$$G(s) = M_s\{\sqrt{\pi}\ 4^{-\lambda}\ (\bar{x})^{A\mu}\ (1+x)^{2\lambda}\ \bar{g}(x)\}$$

$$= \int_0^\infty (\bar{x})^{s-1} [\sqrt{\pi}\ 4^{-\lambda}\ (\bar{x})^{A\mu}\ (1+x)^{2\lambda}\ \bar{g}(x)]\, d\bar{x} \tag{2.6}$$

where the value of $\bar{x}$ is given by the equation (3.7).

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Using the same results (1·1) and (2·2), we also have

$$\frac{2^{2\lambda}}{\sqrt{\pi}}\, \delta^{\mu}\, \dot{M}_s \left\{ \frac{1}{(1+x)^{2}\lambda}\, H^{m,\,n+2}_{n+p+2,\,m+q}\left[\left\{ \frac{ax}{(1+x)^{2}} \right\}^{A} \cdot \delta \,\Big| \right. \right.$$

$$\left. \left. (1-\lambda,\,A),\,(\tfrac{1}{2}-\lambda,\,A),\,[a_n-\mu e_n,\,e_n],\,[\alpha_p-\mu\varepsilon_p,\,\varepsilon_p] \atop [b_m-\mu f_m,\,f_m],\,[\beta_q-\mu\sigma_q,\,\sigma_q] \right] \right\}$$

$$= H^{m,\,n+2}_{n+p+2,\,m+q}\left[\delta \,\Big|\, {(a+s,\,A),\,b-s,\,A),\,[a_n,\,e_n],\,[\alpha_p,\,\varepsilon_p] \atop [b_m,\,f_m],\,[\beta_q,\,\sigma_q]} \right] \qquad (2·7)$$

with the conditions :

(1) m, n, p and q are non negative integers,

(2) $\nu > 0$; value of ν is given by (2·4),

(2) $|\arg.\ \delta\,| < \tfrac{1}{2}\,\pi\,\nu$,

(4) $R(s) < R(\lambda)$,

(5) max. $R\left[\frac{a_k-1}{e_n}\,,\,-\lambda \right] < R(\mu - s) < $ min. $R\left[\frac{b_h}{f_h} \right]$

$$k = 1, 2, \ldots\ldots\ldots, n \ \text{ and }\ R = 1, 2, \ldots\ldots, m.$$

Lastly, here is a property of H-function, of which, we shall make use in the derivation of our result later on.

3. The main result

The aim of this paper is to obtain the following transform pair :

If

$$g(x) = \int_{0}^{\infty} H^{m,\,n+2}_{n+p+2,\,m+q}\left[t \,\Big|\, {(a+ix,A),\,(b-ix,A),\,[a_n,e_n],\,[\alpha_p+\varepsilon_p] \atop [b_m,f_m],\,[\beta_q,\,\sigma_q]} \right] f(t)\, dt \quad (3·1)$$

where $\qquad (m+n) \geqslant p + q - 2.$

then

$$f(x) = \frac{D}{\pi}\int_{-\infty}^{+\infty} \frac{(2\lambda - 2it + s + \mu A - 2)}{\Gamma(2\lambda - it - 2)}\, e^{\pi t}\, g(t)$$

$$\times\ H^{q+1,\,p+1}_{n+p+3,\,m+q+1}$$

$$\left[xe^{\pi i} \,\Big|\, {(\,it-\mu A,\,1),\,[1-\alpha_p-\varepsilon'_p,\,\varepsilon_p],\,[1-a_n+e'_n,\,e_n],\,(\lambda-A,\,A),\,(\tfrac{1}{2}+\lambda-A,\,A) \atop (2\lambda-4-\mu A,\,1),\,[1-\beta_q-\sigma'_q,\,\sigma_q],\,[1-b_m-f'_m,\,f_m]} \right]\, dt \quad (3·2)$$

where $\qquad D = \sqrt{\pi}.\ 4^{1-\lambda-s-\mu A}.\ e^{\pi i(\mu A+1)}$, and $\qquad (3·3)$

$$f'_j = f_j(\mu - 1),\ e'_j = e_j(\mu - 1),\ \sigma'_j = \sigma_j(\mu - 1),\ \varepsilon'_j = \varepsilon_j(\mu - 1) \qquad (3·4)$$

Proof

In order to achieve our aim, let us start from the integral equation (3·1). Substituting $x = -i\,u$, multiplying it by x^{-u} and integrating with respect to u in the limits $c + i\infty$ to $c-i\infty$, we get :

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$$\int_{c-i\infty}^{c+i\infty} x^{-u} \cdot g(-iu)\, du$$

$$= \int_{c-i\infty}^{c+i\infty} x^{-u} \int_0^\infty H^{m,n+2}_{n+p+2,m+q}\left[t \left| \begin{array}{l} (a+u,A),\ (b-u,A)[a_n,e_n],\ [\alpha_p,\varepsilon_p] \\ [b_m,f_m],\ [\beta_q,\sigma_q] \end{array}\right.\right] f(t)\, dt.\, du$$

Putting $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ (3·5)

$$\overline{g}(x) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} x^{-u}\, g(-iu)\, du \qquad\qquad\qquad (3\cdot6)$$

Hence, by virtue of the result (2·7), we have :

$$\overline{g}(x) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} x^{-u}\left[\int_0^\infty \frac{2^{2\lambda}}{\sqrt{\pi}}\cdot t^\mu . M\mu\left\{\frac{1}{(1+x)}2\lambda\times\right.\right.$$

$$H^{m,\,n+2}_{n+p+2,\,m+q}\left[\left(\frac{A}{\overline{x}}\right).t \left| \begin{array}{l}(1-\lambda,A),\ (\tfrac{1}{2}-\lambda,A),\ [a_n-\mu e_n,\,e_n],\ \lceil \alpha_p-\mu\varepsilon_p,\,\varepsilon_p\rceil \\ [b_m-\mu f_m,\,f_m],\ [\beta_q-\mu\sigma_q,\,\sigma_q]\end{array}\right.\right]\Bigg\}\times$$

$$\left.f(t)\, dt\right]du \qquad (3\cdot7)$$

where $\qquad\qquad\qquad\qquad\qquad\qquad \overline{x} = \frac{4x}{(1+x)^2} \qquad\qquad\qquad\qquad\qquad (3\cdot8)$

Changing the order of integration and on simplification, if we invoke (2·7) we get

$$\sqrt{\pi}\cdot 4^{-\lambda}\cdot(\overline{x})^{\mu A}\cdot(1+x)^{2\lambda}\,\overline{g}(x)$$

$$= \int_0^\infty H^{m,\,n+2}_{n+p+2,\,m+q}\left[(\overline{x})^A\cdot t \left| \begin{array}{l} (1-\lambda+\mu A,A),\ (\tfrac{1}{2}-\lambda+\mu A,A),\ [a_n,\,e_n],\ [a_p.\varepsilon_p] \\ [b_m,f_m],\ [\beta_q,\,\sigma_q]\end{array}\right.\right]f(t)\, dt \quad (3\cdot9)$$

Substituting the values of $\overline{x}$ and $\overline{g}(x)$ from (3·8) and (3·6) retpectively replacing s by $1-s$ in the result (2·6), we have, after adjusting the parameters with the help of the result ⌈ (1) ; p. 310 ; (24)].

$$G(1-s) = \frac{\sqrt{\pi}}{4\pi i}\cdot 4^{1-s-\lambda+\mu A}\int_{c-i\infty}^{c+i\infty} e^{-\pi i\,(u+s-\mu A-1)}\times$$

$$(2\lambda-2u-s+\mu A-1)\,\frac{\Gamma(2\lambda+s-\mu A-4)\,\Gamma(\mu A-s-u+1)}{\Gamma(2\lambda-u-2)}\,g(-iu)\,du \qquad (3\cdot10)$$

And in the result (2·5), putting $s = 1-s$, we have :

$$M(1-s) = \frac{\overset{m}{\underset{j=1}{\pi}}\Gamma(b_j-f'_j-f_j s)\ \overset{n}{\underset{j=1}{\pi}}\Gamma(1-a_j+e'_j+e_j s)\,\Gamma(\lambda-A+As)\,\Gamma(\tfrac{1}{2}+\lambda-A+As)}{\overset{q}{\underset{j=1}{\pi}}\Gamma(1-\beta_j+\sigma'_j+\sigma_j s)\ \overset{p}{\underset{j=1}{\pi}}\Gamma(\alpha_j-\varepsilon'_j-\varepsilon_j s)} \qquad (3\cdot11)$$

where e'_j, f'_j, σ'_j and ε'_j are given by (3·4),

Hence, from the results (3·10) and (3·11), we get :

$$f(x) = \frac{\sqrt{\pi}}{(2\pi i)^2}\cdot 4^{1-\lambda-s+\mu A}\int_{c-i\infty}^{c+i\infty} x^{-s}\, ds\int_{c-i\infty}^{c+i\infty} e^{-\pi i(u+s-\mu A-1)}\cdot g(-iu)$$

$$\times\frac{\Gamma(2\lambda+s-\mu A-4)\,\Gamma(\mu A-s-u+1)}{\Gamma(2\lambda-u-2)}(2\lambda-2u+s-\mu A-2)$$

$$\times \quad \frac{\prod\limits_{j=1}^{q} \Gamma(1 - f_j + \sigma'_j + \sigma_j s) \prod\limits_{j=1}^{p} \Gamma(\alpha_j - \varepsilon'_j - \varepsilon_j s)}{\prod\limits_{j=1}^{m} \Gamma(b_j - f'_j - f_j s) \prod\limits_{j=1}^{n} \Gamma(1 - a_j + e'_j + e_j s) \Gamma(\lambda - A + As) \Gamma(\tfrac{1}{2} + \lambda - A + As)} \, du \quad (3\cdot12)$$

or

$$f(x) = \frac{\sqrt{\pi}}{2\pi i} \, 4^{1-\lambda-s+\mu A} \int_{c-i\infty}^{c+i\infty} \frac{(2\lambda - 2u - s + \mu A - 2)}{\Gamma(2\lambda - u - 2)} e^{-\pi i (u - \mu A - 1)}, \ g(-iu)$$

$$\times \ M^{\ 1}_{xe^{\pi i}} \left\{ \frac{\prod\limits_{j=1}^{q} \Gamma(1 - \beta_j + \sigma'_j + \sigma_j s) \, \Gamma(2\lambda - 4 - \mu A + s) \prod\limits_{j=1}^{p} \Gamma(\alpha_j - \varepsilon'_j - \varepsilon_j s) \, \Gamma(\mu A + 1 - u - s)}{\prod\limits_{j=1}^{m} \Gamma(b_j - f'_j - f_j s) \prod\limits_{j=1}^{n} \Gamma(1 - a_j + e'_j + e_j s) \, \Gamma(\lambda - A + As) \, \Gamma(\tfrac{1}{2} - \lambda - A + As)} \right\} du$$

$$(3\cdot13)$$

Therefore by virtue of the result $(2\cdot2)$, we get :

$$f(x) = \frac{D}{2\pi i} \int_{c-i\infty}^{c+i\infty} \frac{(2\lambda - 2u - s + \mu A - 2)}{\Gamma(2\lambda - u - 2)} \, e^{-\pi i u} \cdot g(-iu)$$

$$\times \ H^{q+1,\ p+1}_{n+p+3,\ m+q+1} \left[\ xe^{\pi i} \ \middle| \ \begin{array}{l} (u - \mu A, 1), [1 - \alpha_p - \varepsilon'_p, \varepsilon_p], [1 - a_n + e'_n, e_n], \\ (2\lambda - 4, \mu A, 1), [1 - \beta_q + \sigma'_q, \sigma_q] \end{array} \right. \times$$

$$\left. \begin{array}{c} (\lambda - A, A), (\tfrac{1}{2} + \lambda - A, A) \\ [1 - b_m - f'_m, f_m] \end{array} \right] du \qquad (3\cdot14)$$

where D is given by $(3\cdot5)$.

Putting $c = 0$ in $(3\cdot14)$, we get the required result $(3\cdot2)$, viz. :

$$f(x) = \frac{D}{\pi} \int_{-\infty}^{+\infty} \frac{2\lambda - 2i\,t + s + \mu A - 2}{\Gamma(2\lambda - it - 2)} \, e^{\pi t} \, g(t)$$

$$\times \ H^{q+1,\ p+1}_{n+p+3,\ m+q+1} \left[\ xe^{\pi i} \ \middle| \ \begin{array}{l} (it - \mu A, 1), [1 - \alpha_p - \varepsilon'_p, \varepsilon_p], [1 - a_n + e'_n, e_n] \\ (2\lambda - 4, \mu A, 1), [1 - \beta_q + \sigma'_q, \sigma_q], \end{array} \right. \times$$

$$\left. \begin{array}{c} [\lambda - A, A], [\tfrac{1}{2} + \lambda - A, A] \\ [1 - b_m - f'_m, f_m] \end{array} \right] dt \qquad (3\cdot15)$$

where D is given by $(3\cdot5)$.

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Studies on the effect of various levels of nitrogen at different times of application on the seed yield, oil content and quantity of yellow sarson (*B compestris*)

By

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Oil seeds occupy an important place as a good source of foreign currency. An area of 1·6 thousand hectares is under cultivation of rape and mustard crop in Uttar Pradesh. Studies on breeding of yellow sarson for higher seed yields have been conducted in the past. But, little published evidence is at hand, on the effects of the time of application of fertilizers it's relationship with plant characters, yield and the quality of oil. .

Sinha *et al.* (1961) observed increased protein content and inverse relationship between protein and oil content in yellow sarson plants where nitrogenous fertilizers were used. They also reported in the case of rai, an uptake of nitrogen upto maturity and depressions in the oil percentage due to nitrogenous manuring. Mehrotra *et al.* (1967) noticed a progressive nitrogen and phosphorus uptake upto the maturity of the crop. Sen and Sarkar (1958) found significant increases in mustard yield with phosphate manuring. S n and Lahiri (1960) also reported increased dry matter and seed yield in sarson with phosphate. The present investigations were taken up with a view to study the influence of nitrogenous fertilizers, applied at different growth phases of yellow sarson, on general morphology of the crop seed and oil yield and quantity of oil.

Materials and Methods

The experiment was laid out in medium loam soil of PIRRCOM Farm, Kalianpur, Kanpur. The lay out was randomized block design with eleven treatments and four replications. Three levels of nitrogen (0, 30 and 60 lb/ac.) were applied at different stages.

Levels of N		*Time of Application*
N_0 – No nitrogen		T_0 – No application of fertiliser
N_1 – 30 lb N/ac.	X	T_1 – Full dose at sowing.
N_2 – 60 lb N/ac.		T_2 – Full dose at one month stage.
		T_3 – $\frac{1}{2}$ at sowing + $\frac{1}{2}$ at one month.
		T_4 – $\frac{1}{2}$ at sowing + $\frac{1}{2}$ at two months.
		T_5 – $\frac{1}{2}$ at one month + $\frac{1}{2}$ at two months.

Important plant characters, *viz.*, height of the plant, number of branches per plant, number of pods per plant and seed yield were recorded after the harvest of the crop. Seed samples of yellow sarson were also collected at the time of harvesting of the crop in polythene bags. These samples were cleaned and made free of

all extraneous matters and stored in airtight bottles for chemical analyses in the laboratory.

. Thr morphological and yield data are given in Table 1 and oil content and its quality are given in Table 2.

TABLE 1

Seed yield and plant characters of yellow sarson as influenced by the time of nitrogen application

Treatments	Height plant in cms.	No. of branches/ plant	No. of pods plant	Yield in kgms/ acre
A $N_1 T_1$	148·00	5·50	106·85	579·62
B $N_1 T_2$	141·22	5·17	94·00	631·49
C $N_1 T_3$	141·30	4·47	81·85	625·69
D $N_1 T_4$	147·95	4·72	90·20	617·96
E $N_1 T_5$	138·50	4·95	83·15	579·94
F $N_2 T_1$	153·90	7·22	147·17	751·35
G $N_2 T_2$	143·05	6·15	107·10	771·65
H $N_2 T_3$	154·07	7·32	145·82	772·93
I $N_2 T_4$	143·22	6·80	121·22	697·86
J $N_2 T_5$	135·75	5·45	92·45	741·04
K $N_0 T_0$	129·65	2·87	58·67	337·33
'F' test	Sign. at 1%	Sign. at 1%	Sign. at 1%	Sign at 1%
S. E. M. ±	4·4	0·51	11·02	–
C. D. at 5%	12·67	–	–	144·739
C. D. at 1%	–	1·98	42·76	194·923

TABLE 2

Effect of various levels of nitrogen at different times of application on the content and quality of oil of yellow sarson.

Treatments	Oil %	Proteins %	Allylis- othiocy- nate value	Iodine value	Acid value	Free fatty acid
A 30 lb. N/acre full at sowing	46·3	5·34	0·27	107·2	2·09	1·05
B 30 lb. N/acre at one month	46·3	5·64	0·26	99·0	2·25	1·13
C 30 lb N/acre ½ at sowing + ½ at one month	46·2	5·99	0·26	105·0	1·60	0·808
D 30 lb. N/acre ½ at sowing + ½ at 2 months.	44·9	5·95	0·27	105·0	1·70	0·808
E 30 lb. N/acre ½ at one month + ½ at 2 months	43·9	5·95	0·28	106·4	1·77	0·89
F 60 lb. N/acre full at sowing	45·1	5·64	0·27	97·6	2·09	1·05
G 60 lb. N/acre at one month	45·0	5·67	0·28	99·0	1·77	0·89
H 60 lb. N/acre ½ at sowing + ½ at one month	44·7	5·82	0·28	100·5	1·93	0·97
I 60 lb. N/acre ½ at sowing + ½ at two months	44·6	5·87	0·27	104·2	1·77	0·89
J 60 lb. N/acre ½ at one month + ½ at 2 months	44·3	5 95	0·28	102·0	1·60	0 81
K Control	46·3	5·32	0·25	105·0	2·25	1·13

Results and Discussion

Morphological Characters and seed yield :

The nitrogen application was found to be very effective in promoting all growth and yield contributing factors.

Occular differences in the growth and development of the plants were marked due to nitrogenous fertilizers and all the plant characters under study were significantly altered by nitrogen application. The results obtained with regard to seed yield were in line with growth characters. The average responsiveness of nitrogen at N_{30} was seen to be 269·61 Kg/ac which was further augmented to 409·63 Kg./ac in case of N_{60}. These responses ultimately showed an increase yield of about 80 and 121% respectively over control. No marked differences in yield due to nitrogenous manuring were observed.

Oil content and it's quality : The application of nitrogen at 30 lb. N/acre full at sowing, or one month after and half at sowing and the other half one month after did not depress the oil content, whereas, in other treatments at 30 or 60 lb. N/per acre the oil content was appreciably depressed. The decrease in oil content was comparatively higher in treatments where nitrogen was given two months after sowing. This may be due to the fact that the oil formation starts on fifth day after flowering (Sinha and Agarwal, 1963) and the application of nitrogen at late stages might be hampering with the formation of oil adversely. There was a positive correlation between nitrogenous manuring and protein content. Allyliso-thiocynate value has also increased slightly in comparison to unmanured seeds.

Summary and Conclusions

1. The yield of yellow sarson seed (B. Compestris) responded well to higher doses of manuring.

2. The oil content was depressed due to nitrogenous manuring at higher doses.

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Studies on nutrition of Indian cereals—III. The uptake of phosphorus by wheat plants at different growth phases in relation to fertilizer application

By

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Introduction

Experimental evidence with regard to effects of nitrogenous and phosphatic fertilizers on the composition and uptake of phosphorus by wheat plants is somewhat contradictory. Larson *et al.* (1952) reported increased percentage of phosphorus in wheat plants with phosphatic fertilizers, while De Turk (1942), working on phosphate deficient soils of Illinois, observed decreased phosphorus content in wheat plants due to phosphorus application. Similarly, depressive effect of nitrogen on phosphorus concentration of wheat has been reported by Sharma (1962) in contrast to the findings of Rennie and Soper (1958) and Singh (1962), who noted beneficial effect of nitrogen on phosphorus absorption. Combined application of nitrogen and phosphorus has, however, been shown to be superior in promoting the absorption of these nutrients by the plants as compared with their single application (Arakeri *et al.*, 1961 and Bolaria and Mann, 1964).

In an earlier paper the authors (1967) have reported the results of the uptake of nitrogen by wheat plant at various stages of growth as influenced by phosphorus. The present communication deals with the pattern of uptake of phosphorus by wheat at different growth phases in relation to fertilizer application.

Materials and Methods

The present study was undertaken in 1962-63 on wheat var. N. P. 710 grown at the Experimental Station, Dilkusha Farm, Lucknow. The layout adopted was randomized block design with 9 treatments and 4 replications. Three levels of nitrogen, *i. e.*, 0, 28 and 56 Kg N/ha (N_0, N_1 and N_2), as sulphate of ammonia, were applied alone or in combination with three doses of phosphorus, *i. e.*, 0, 22·4 and 44·8 Kg P_2O_5/ha (P_0, P_1 and P_2) given as single superphosphate. The soil was sandy loam with a moderate fertility and average 7·3 pH.

Wheat plant samples (above ground portion only) were collected from each plot and composite samples were made according to each treatment. The samplings were done at successive growth stages of the crop, *viz.*, seedling, active tillering, jointing, flag leaf, earning, ripening and harvest. These samples were dried at a temperature of 105°C for 48 hours and ground in a laboratory mill. Phosphorus content of plant samples was determined according to the method described by Snell and Snell (1949) using Spekker's Absorptiometer. Uptake of phosphorus was calculated on dry matter production basis and recovery of added phosphorus evaluated by Donee's formula (1934).

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Results

The phosphorus concentration, dry matter content and uptake of phosphorus by wheat at different phases of growth are shown in Tables 1, 2 and 3 respectively.

1. *Phosphorus concentration in wheat plant as affected by levels of fertilizers.*

The absorption of phosphorus by wheat plant, irrespective of treatments, started from the seedling stage of the crop, was maximum at active tiller development stage and gradually declined with ontogeny. Application of phosphates improved the phosphorus content in plants, whereas nitrogenous fertilizers alone slightly depressed it. A combined dressing of nitrogen and phosphorus was more beneficial with regard to concentration of phosphorus, which was maximum in plants treated with 56 Kg N + 44·8 Kg P_2O_5/ha.

TABLE 1

Total phosphorus (% dry weight basis) in wheat plants as affected by various levels of nitrogenous and phosphatic fertilizers

Treatments Kg/ha	Seedling	Tillering	Growth Stages Jointing	Flag leaf	Earing	Ripening	Harvest
N_0P_0	0·2750	0·4500	0·4125	0·4000	0·3725	0·3400	0·3000
N_0P_1	0·3000	0·4500	0·4175	0·4062	0·3775	0·3425	0·3075
N_0P_2	0·3125	0·4575	0·4200	0·4185	0·3825	0·3425	0·3100
N_1P_0	0·2500	0·4475	0·4075	0·3900	0·3675	0·3370	0·3000
N_1P_1	0·3250	0·4875	0·4250	0·4250	0·3900	0·3500	0·3150
N_1P_2	0·3750	0·5000	0·4350	0·4375	0·3975	0·3575	0·3275
N_2P_0	0·2700	0·4375	0·4100	0·3825	0·3650	0·3370	0·2975
N_2P_1	0·3900	0·5425	0·4450	0·4435	0·4020	0·3620	0·3320
N_2P_2	0·4000	0·5500	0·4550	0·4475	0·4070	0·3695	0·3395

TABLE 2

Dry matter production of wheat plants (per plant in Gms.)

Treatment Kg/ha	Seedling	Tillering	Growth Stages Jointing	Flag leaf	Earing	Ripening	Harvest
N_0P_0	0·082	2·49	2·98	3·20	8·42	10·00	10·50
N_0P_1	0·083	2·51	3·01	3·41	0·98	10·45	11·00
N_0P_2	0·085	2·52	3·24	3·62	9·14	11·00	11·35
N_1P_0	0·087	2·68	3·30	3·78	9·88	11·75	12·00
N_1P_1	0·090	2·76	3·90	4·05	10·23	12·51	13·00
N_1P_2	0·091	2·93	3·98	4·19	10·58	13·00	13·70
N_2P_0	0·104	3·04	3·99	4·53	11·72	13·87	14·95
N_2P_1	0·106	3·06	4·48	4·87	11·75	14·25	15·27
N_2P_2	0·110	3·13	4·49	5·04	12·00	14·98	17·25

2. Uptake of phosphorus at different growth stages of wheat

The data portrayed in Table 3 reveal a steady uptake of phosphorus from seedling to harvest stages of the crop in all the treatments. Fertilizing the crop with nitrogen or phosphorus was associated with an increased phosphorus uptake which was more pronounced when combined dressings of both the fertilizers were done. Highest uptake (19 Kg phosphorus per hectare) was observed at maturity where 56 Kg N + 44·8 Kg P_2O_5 per hectare was applied. Of the total phosphorus absorbed by the plant, about 70% was taken upto the earing stage.

TABLE 3

Uptake of phosphorus by wheat plants at different phases of growth and yield of grain (Kg/ha)

Treatment, Kg/ha	Seedling	Tillering	Jointing	Flag leaf	Earing	Ripening	Harvest	Yield of of Grain
N_0P_0	0·065	3·237	3·546	3·712	9·084	9·900	11·760	1894
N_0P_1	0·072	3·263	3·642	4·024	9·788	10·345	12·430	1819
N_0P_2	0·076	3·326	3·953	4·380	10·054	10·890	13·052	1792
N_1P_0	0·063	3·457	3·894	4·272	10·473	11·515	13·320	2311
N_1P_1	0·085	3·892	4·797	4·981	11·560	12·635	15·210	2387
N_1P_2	0·098	4·248	5·015	5·321	11·167	13·520	16·303	2473
N_2P_0	0·081	3·830	4·748	5·082	12·306	13·593	16·445	2728
N_2P_1	0·120	4·804	5·779	5·912	13·630	14·962	17·713	2766
N_2P_2	0·128	4·977	5·927	6·552	14·160	16·029	20·355	3145

Discussion

Absorption of phosphorus by wheat plants in the present study was highest at the tillering stage, which is in agreement with the results reported by Boatwright and Haas (1961) and Bolaria and Mann (1964) in wheat. The decrease in phosphorus concentration after the tillering stage may be attributed to the progressive dilution of phosphorus by CHO, which is associated with the maturation of the crop (Dougall, 1963). Increased percentage of phosphorus due to superphosphate application has also been reported by Larson et al. and Arakeri et al. (loc. cit.). Higher phosphorus content of plants fertilized with combined N and P might have resulted from an increase in acidity caused by ammonium sulphate, which in turn reduced the rate of fixation of the phosphorus fertilizers (Rennie and Mitchell, 1954).

The phosphorus uptake continued from seedling stage to harvest regardless of combined fertilizer applications. This is in confirmity with the results obtained by Mattingly and Widdowson (1958) in wheat. Stimulated uptake of phosphorus caused by application of nitrogen alone was possibly due to more dry matter production. Considerable increase in the uptake of phosphorus in plants where nitrogen and phosphorus were applied together might be due to stimulated top and root growth and a decreased pH and consequently an increased uptake of fertilizer phosphorus (Grunes et al., 1958).

A perusal of data summarised in Tables 1, 2 and 3 show that the uptake of phosphorus by wheat plants appears to be mo e a function of the increases in dry matter production than the increases in phosphorus content of the plants, which is in accordance with the findings of Bennett *et al.* (1953).

Increases in doses of phosphorus when applied alone were associated with a corresponding decrease in the percent recovery of phosphorus applied through fertilizer, which was 3·45 and 2·94 at 32·4 Kg and 44·8 Kg. P_2O_5 per hectare respectively. Application of nitrogen alongwith phosphatic fertilizers enhanced the percent recovery of phosphorus. The best recovery of phosphorus (27%) was observed in treatment, where 56 Kg N + 44·8 Kg P_2O_5 per hectare were applied together.

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On some results involving Generalized Hypergeometric and Gegenbauer (Ultraspherical) Polynomials

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Abstract

The object of this paper is to obtain some results involving the generalized hypergeometric and Gegenbauer polynomials by defining the polynomial in the form

$$F_n(x) = x^{(m-1)n} \; {}_{p+m}F_q \left[\begin{array}{c} \Delta \, (m, \, -n), \, a_1, \, \ldots \ldots, \, a_p \\ b_1, \, \ldots \ldots, \, b_q \end{array} ; \, \lambda x^c \right]$$

where $\Delta \, (m, -n)$ represents the set of m–parameters :

$$\frac{-n}{m}, \; \frac{-n+1}{m}, \; \ldots \ldots, \; \frac{-n+m-1}{m}$$

and m, n are positive integers. The polynomial is in a generalized form which yields many known polynomials by particular choice of parameters. A number of known and new results are also given.

1. Introduction

We have defined the generalized hypergeometric polynomial [(7), p. 79, eqn. (2·1)] by means of

$$(1\cdot1) \qquad F_m(x) = x^{(\delta-1)m} \; {}_{p+\delta}F_q \left[\begin{array}{c} \Delta \, (\delta, \, -m), \, a_1, \, \ldots \ldots, \, a_p \\ b_1, \, \ldots \ldots, \, b_q \end{array} ; \, \mu x^c \right]$$

where m and δ are positive integers and the symbol $\Delta \, (\delta, -m)$ denotes the set of δ – parameters :

$$\frac{-m}{\delta}, \; \frac{-m+1}{\delta}, \; \ldots \ldots, \; \frac{-m+\delta-1}{\delta}.$$

This polynomial has arisen in the course of an attempt to unify and to extend the study of most of the well-known sets of polynomials.

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In what follows for sake of brevity a_p stands for $a_1, \ldots \ldots \ldots, a_p; (a_p)_r$

denotes $\overset{p}{\underset{j=1}{\pi}} (a_j)_r$ and similarly for $(b_q)_r$.

In this paper we have established some results of integrals involving the product of generalized hypergeometric and Gegenbauer (Ultraspherical) polynomials. Some expansion formulae for generalized hypergeometric polynomials have been derived with the help of these integrals. Many known and new results have also obtained on specializing the parameters. Therefore the results obtained in this paper are of general character.

2. Integrals :

In this section we have evaluated some integrals involving generalized hypergeometric and Gegenbauer polynomials.

The integrals to be evaluated are

$$(2 \cdot 1) \qquad \int_0^1 x^\lambda (1-x)^{\nu-\frac{1}{2}} C_n^\nu (2x-1) \left\{ x^{(\delta-1)m} {}_{p+\delta}F_q \left[\begin{matrix} \triangle(\delta,-m), a_p \\ b_q \end{matrix} ; \mu x^c \right] \right\} dx$$

$$= \frac{\Gamma(\nu+\frac{1}{2}) \Gamma(2\nu+n) \Gamma(\lambda+(\delta-1)m+1) \Gamma(\lambda+(\delta-1)\frac{m-\nu+\frac{3}{2}}{m+\nu+n+\frac{3}{2}})}{n! \Gamma(2\nu) \Gamma(\lambda+(\delta-1)m-\nu-n+\frac{3}{2}) \Gamma(\lambda+(\delta-1)\frac{m-\nu+\frac{3}{2}}{m+\nu+n+\frac{3}{2}})}$$

$$\times {}_{p+\delta+2c} F_{q+2c} \left[\begin{matrix} \triangle(\delta,-m), a_p, \triangle(c, \lambda+(\delta-1)m+1), \triangle(c, \lambda+(\delta-1)m-\nu+\frac{3}{2}) \\ b_q, \triangle(c, \lambda+(\delta-1)m-\nu-n+\frac{3}{2}), \triangle(c, \lambda+(\delta-1)m+\nu+n+\frac{3}{2}) \end{matrix} : \mu \right]$$

where δ, m and c are positive integers, $Re(\lambda+(\delta-1)m) > -1$, and $Re(\nu) > -\frac{1}{2}$.

$$(2 \cdot 2) \qquad \int_0^1 x^\lambda (1-x)^{\nu-\frac{1}{2}} C_n^\nu (2x-1) \left\{ x^{(\delta-1)m} {}_{p+\delta}F_q \left[\begin{matrix} \triangle(\delta,-m), a_p \\ b_q \end{matrix} ; \mu x^{-c} \right] \right\} dx$$

$$= \frac{\Gamma(\nu+\frac{1}{2}) \Gamma(2\nu+n) \Gamma(\lambda+(\delta-1)m+1) \Gamma(\lambda+(\delta-1)\frac{m-\nu+\frac{3}{2}}{m-\nu-n+\frac{3}{2}})}{n! \Gamma(2\nu) \Gamma(\lambda+(\delta-1)m-\nu-n+\frac{3}{2}) \Gamma(\lambda+(\delta-1)\frac{m+\nu+n+\frac{3}{2}}{m+\nu+n+\frac{3}{2}})}$$

$$\times {}_{p+\delta+2c} F_{q+2c} \left[\begin{matrix} \triangle(\delta,-m), a_p, \triangle(c, -\lambda-(\delta-1)m+\nu+n-\frac{1}{2}), \triangle(c, -\lambda-(\delta-1)m-\nu-n-\frac{1}{2}) \\ b_q, \triangle(c, -\lambda-(\delta-1)m), \triangle(c, -\lambda-(\delta-1)m+\nu-\frac{1}{2}) \end{matrix} : \mu \right]$$

valid for $Re(\lambda+(\delta-1)m) > -1$, $Re(\nu) > -\frac{1}{2}$, m, c and δ are positive integers.

Proofs :

(A) To prove (2·1), we write down the eries for the generalized hypergeometric polynomial in the integrand, change the order of integration and summation which we suppose to be permissible due to the absolute convergence of the integral and summation involved in the process, we obtain

$$\sum_{r=0}^\infty \frac{\overset{\delta-1}{\underset{i=0}{\pi}} \left(\frac{-m+i}{\delta} \right)_r (a_p)_r \mu^r}{r! (b_q)_r} \int_0^1 x^{\lambda+(\delta-1)m+cr} (1-x)^{\nu-\frac{1}{2}} C_n^\nu (2x-1) \, dx.$$

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Now evaluating the integral with the help of the known result [(1), p. 280,(3)]:

$$(2\cdot3) \qquad \int_0^1 x^\lambda\,(1-x)^{\nu-1}\,C_n^\nu\,(2x-1)\,dx$$

$$= \frac{\Gamma(\lambda+1)\,\Gamma(\nu+\tfrac{1}{2})\,\Gamma(2\nu+n)\,\Gamma(\lambda-\nu+\tfrac{3}{2})}{!\,\Gamma(2\nu)\,\Gamma(\lambda-\nu-n+\tfrac{3}{2})\,\Gamma(\lambda+\nu+n+\tfrac{3}{2})}\quad,$$

valid for $Re(\lambda) > -1$, $Re(\nu) > -\tfrac{1}{2}$ and using the following relations

$$(\alpha)_n = \frac{\Gamma(\alpha+n)}{\Gamma(\alpha)}, \text{ and } (\alpha)_{nk} = k^{nk}\prod_{i=0}^{k-1}\left(\frac{\alpha+i}{k}\right)_n,$$

we have

$$\frac{\Gamma(\nu+\tfrac{1}{2})\,\Gamma(2\nu+n)\,\Gamma(\lambda+(\delta-1)\,m+1)\,\Gamma(\lambda+(\delta-1)\,m-\nu+\tfrac{3}{2})}{n!\,\Gamma(2\nu)\,\Gamma(\lambda+(\delta-1)\,m-\nu-n+\tfrac{3}{2})\,\Gamma(\lambda+(\delta-1)\,m+\nu+n+\tfrac{3}{2})}$$

$$\sum_{r=0}^{\infty}\frac{\prod\limits_{i=0}^{\delta-1}\left(\dfrac{-m+i}{\delta}\right)_r (a_p)_r \prod\limits_{i=0}^{c-1}\left(\dfrac{\lambda+(\delta-1)\,m+1+i}{c}\right)_r \prod\limits_{i=0}^{c-1}\left(\dfrac{\lambda+(\delta-1)\,m-\nu+\tfrac{3}{2}+i}{c}\right)_r \mu^r}{r!\,(b_q)_r \prod\limits_{i=0}^{c-1}\left(\dfrac{\lambda+(\delta-1)\,m-\nu-n+\tfrac{3}{2}+i}{c}\right)_r \prod\limits_{i=0}^{c-1}\left(\dfrac{\lambda+(\delta-1)\,m+\nu+n+\tfrac{3}{2}+i}{c}\right)_r}$$

which yields the value of the integral (2·1).

(B) Integral (2·2) may be derived on applying the same procedure as above and after using (2·3) and following relations

$$(\alpha)_n = \frac{\Gamma(\alpha+n)}{\Gamma(\alpha)}\ ,\quad \frac{\Gamma(1-\alpha-n)}{\Gamma(1-\alpha)} = \frac{(-1)^n}{(\alpha)_n}\ ,\quad (\alpha)_{nk} = k^{nk}\prod_{i=0}^{k-1}\left(\frac{\alpha+i}{k}\right)_n,$$

we get

$$\frac{\Gamma(\nu+\tfrac{1}{2})\,\Gamma(2\nu+n)\,\Gamma(\lambda+(\delta-1)\,m+1)\,\Gamma(\lambda+(\delta-1)\,m-\nu+\tfrac{3}{2})}{n!\,\Gamma(2\nu)\,\Gamma(\lambda+(\delta-1)\,m-\nu-n+\tfrac{3}{2})\,\Gamma(\lambda+(\delta-1)\,m+\nu+n+\tfrac{3}{2})}$$

$$\times \sum_{r=0}^{\infty}\frac{\prod\limits_{i=0}^{\delta-1}\left(\dfrac{-m+i}{\delta}\right)_r (a_p)_r \prod\limits_{i=0}^{c-1}\left(\dfrac{-\lambda-(\delta-1)m+\nu+n-\tfrac{1}{2}+i}{c}\right)_r}{r!\,(b_q)_r \prod\limits_{i=0}^{c-1}\left(\dfrac{-\lambda-(\delta-1)\,m+i}{c}\right)_r}$$

$$\frac{\prod\limits_{i=0}^{c-1}\left(\dfrac{-\lambda-(\delta-1)\,m-\nu-n-\tfrac{1}{2}+i}{c}\right)_r \mu^r}{\prod\limits_{i=0}^{c-1}\left(\dfrac{-\lambda-(\delta-1)\,m+\nu-\tfrac{1}{2}+i}{c}\right)_r}$$

which is the right hand side of (2·2).

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3. Expansions

This section is concerned with the expansion formulas for the generalized hypergeometric polynomials in series of Gegenbauer polynomials which have been derived with the application of the integrals evaluated in section 2.

Expansion formulas to be established are

$$(3\cdot1) \qquad x^{\lambda}\left\{x^{(\delta-1)m}\,{}_{p+\delta}F_q\left[\begin{array}{c}\triangle(\delta,-m),\ a_p \\ b_q\end{array};\ \mu\,x^c\right]\right\}$$

$$=\frac{2^{2\nu}\,\Gamma(\nu)\,\Gamma(\lambda+\nu+(\delta-1)\,m+\tfrac{1}{2})}{\sqrt{\pi}\ \Gamma(\lambda+2\nu+(\delta-1)\,m+1)}\ \sum_{r=0}^{\infty}\frac{(-1)^r\,(\nu+r)\,(-\lambda-(\delta-1)\,m)_r}{(\lambda+2\nu+(\delta-1)\,m+1)_r}$$

$$\times\ {}_{p+\delta+2c}F_{q+2c}\left[\begin{array}{c}\triangle(\delta,-m),\ a_p,\ \triangle(c,\ \lambda+\nu+(\delta-1)\,m+\tfrac{1}{2}),\ \triangle(c,\ \lambda+(\delta-1)\,m+1) \\ b_q,\ \triangle(c,\ \lambda+(\delta-1)\,m-r+1),\ \triangle(c,\ \lambda+2\nu+(\delta-1)\,m+r+1)\end{array};\ \mu\right]$$

$$C_r^{\nu}\,(2x-1)$$

valid for $R(\lambda+(\delta-1)\,m)>-\tfrac{1}{2}$, $Re(\nu)>0$, δ, m and c are positive integers.

$$(3\cdot2) \qquad x^{\lambda}\left\{x^{(\delta-1)m}\,{}_{p+\delta}F_q\left[\begin{array}{c}\triangle(\delta,\ -m),\ a_p \\ b_q\end{array};\ \mu x^c\right]\right\}$$

$$=\frac{2^{2\nu}\,\Gamma(\nu)\,\Gamma(\lambda+\nu+(\delta-1)\,m+\tfrac{1}{2})}{\sqrt{\pi}\ \Gamma(\lambda+2\nu+(\delta-1)\,m+1)}\ \sum_{r=0}^{\infty}\frac{(-1)^r\,(\nu+r)\,(-\lambda-(\delta-1)\,m)_r}{(\lambda+2\nu+(\delta-1)\,m+1)_r}$$

$$\times\ {}_{p+\delta+2c}F_{q+2c}\left[\begin{array}{c}\triangle(\delta,-m),\ a_p,\ \triangle(c,-\lambda-(\delta-1)\,m+r),\ \triangle(c,-\lambda-2\nu-(\delta-1)\,m-r) \\ b_q,\ \triangle(c,-\lambda-\nu+\tfrac{1}{2}-(\delta-1)\,m),\ \triangle(c,-\lambda-(\delta-1)\,m)\end{array};\ \mu\right]$$

$$C_r^{\nu}\,(2x-1)$$

where $R(\lambda+(\delta-1)\,m)>-\tfrac{1}{2}$, $Re(\nu)>0$, δ, m and c are positive integers.

Proof :

Let

$$(3\cdot3) \qquad \begin{cases} f(x)=x^{\lambda}\left\{x^{(\delta-1)m}\,{}_{p+\delta}F_q\left[\begin{array}{c}\triangle(\delta,-m),\ a_p \\ b_q\end{array};\ \mu\,x^c\right]\right\}=\sum\limits_{r=0}^{\infty}A_r C_r^{\nu}\,(2x\ 1) \\[3mm] C_r^{\nu}\,(2x-1)=\dfrac{(2\nu)_r}{r!}\ {}_2F_1\left(\begin{array}{c}-r,\ r+2\nu \\ \nu+\tfrac{1}{2}\end{array};\ 1-x\right), \end{cases}$$

$$(0<x<1).$$

Here $C_r^{\nu}\,(2x-1)$ is a Gegenbauer polynomial [(6), p. 279, (15)]. Equation (3·3) is valid since $f(x)$ is continuous and of bounded variation in the open interval (0, 1). Now multiply both sides of (3·3) by $x^{\nu-\frac{1}{2}}\,(1-x)^{\nu-\frac{1}{2}}\,C_n^{\nu}\,(2x-1)$ and integrate with respect to x from 0 to 1. Change the order of integration and summation which is easily seen to be justified on the right, we have

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$$(3\cdot4) \quad \int_0^1 x^{\lambda+\nu-\frac{1}{2}}(1-x)^{\nu-\frac{1}{2}} C_n^\nu(2x-1)\left\{x^{(\delta-1)m}\ _{p+\delta}F_q\left[\begin{array}{c}\triangle(\delta,-m),\ a_p\\ b_q\end{array};\mu x^c\right]\right\}dx$$

$$=\sum_{r=0}^\infty A_r \int_0^1 x^{\nu-\frac{1}{2}}(1-x)^{\nu-\frac{1}{2}} C_n^\nu(2x-1) C_r^\nu(2x-1)\ dx$$

where $Re(\lambda+\nu+(\delta-1)m) > -\frac{1}{2}$, $Re(\nu) > -\frac{1}{2}$.

Using the orthogonality property for the Gegenbauer polynomials [(6), p. 281, (28)]:

$$\int_0^1 x^{\nu-\frac{1}{2}}(1-x)^{\nu-\frac{1}{2}}\left[C_n^\nu(2x-1)\right]^2 dx = \frac{\sqrt{\pi}\ \Gamma(2\nu+n)\ \Gamma(\nu+\frac{1}{2})}{2^{2\nu}\ n!\ (\nu+n)\ \Gamma(2\nu)\ \Gamma(\nu)},$$

valid for $Re(\nu) > -\frac{1}{2}$,

on the right and the result (2·1) on the left of (3·4), we obtain.

$$(3\cdot5)\quad A_n = \frac{2^{2\nu}(\nu+n)\ \Gamma(\nu)\ \Gamma(\lambda+\nu+(\delta-1)m+\frac{1}{2})\ \Gamma(\lambda+(\delta-1)m+1)}{\sqrt{\pi}\ \Gamma(\lambda+(\delta-1)m-n+1)\ \Gamma(\lambda+2\nu+(\delta-1)m+n+1)}$$

$$\times\ _{p+\delta+2c}F_{q+2c}\left[\begin{array}{c}\triangle(\delta,-m),\ a_p,\ \triangle(c,\lambda+\nu+(\delta-1)m+\frac{1}{2}),\ \triangle(c,\lambda+(\delta-1)m+1)\\ b_q,\ \triangle(c,\lambda+(\delta-1)m-n+1),\ \triangle(c,\lambda+2\nu+(\delta-1)m+n+1)\end{array};\mu\right]$$

where $Re(\nu) > 0$, $Re(\lambda+(\delta-1)m) > -1$.

With the help of (3·3) and (3·5), we obtain the expansion formula (3·1).

The expansion formula (3·2) is similarly established on the same lines as above and using the result (2·2).

4. Applications :

In this section we have considered a number of particular cases of the integrals and expansion formulas established in sections 2 and 3.

(A) Special cases of (2·1) and (3·1) with $\delta = \mu = c = 1$:

(a) Taking $a_1 = m+\alpha+\beta+1$, $b_1 = 1+\alpha$, $b_2 = \frac{1}{2}$ and multiplying both sides by $\frac{(1+\alpha)_m}{m!}$, we obtain

$$(4\cdot1)\quad \int_0^1 x^\lambda(1-x)^{\nu-\frac{1}{2}} C_n^\nu(2x-1)\ f_m^{(\alpha,\beta)}\left(\begin{array}{c}a_2,\ \ldots,\ a_p\\ b_3,\ \ldots,\ b_q\end{array};x\right)dx$$

$$=\frac{\Gamma(\nu+\frac{1}{2})\ \Gamma(2\nu+n)\ \Gamma(\lambda+1)\ \Gamma(\lambda-\nu+\frac{3}{2})}{n!\ \Gamma(2\nu)\ \Gamma(\lambda-\nu-n+\frac{3}{2})\ \Gamma(\lambda+\nu+n+\frac{3}{2})}$$

$$\times\ f_m^{(\alpha,\beta)}\left(\begin{array}{c}a_2,\ \ldots,\ a_p,\ \lambda+1,\ \lambda-\nu+\frac{3}{2}\\ b_3,\ \ldots,\ b_q,\ \lambda-\nu-n+\frac{3}{2},\ \lambda+\nu+n+\frac{3}{2}\end{array};1\right)$$

valid for $Re(\lambda) > -1$, $Re(\nu) > -\frac{1}{2}$.

$(4\cdot2)$
$$x^\lambda \, f_m^{(\alpha,\,\beta)} \left(\begin{matrix} a_2, \ldots\ldots, a_p \\ b_3, \ldots\ldots, b_q \end{matrix} ; x \right)$$

$$= \frac{2^{2\nu}\,\Gamma(\nu)\,\Gamma(\lambda+\nu+\tfrac{1}{2})}{\sqrt{\pi}\ \Gamma(\lambda+2\nu+1)} \sum_{r=0}^{\infty} \frac{(-1)^r\,(\nu+r)\,(-\lambda)_r}{(\lambda+2\nu+1)_r}$$

$$\times f_m^{(\alpha,\beta)} \left(\begin{matrix} a_2, \ldots\ldots, a_p, \lambda+\nu+\tfrac{1}{2}, \lambda+1 \\ b_3, \ldots\ldots, b_q, \lambda-r+1, \lambda+2\nu+r+1 \end{matrix} ; 1 \right) C_r^\nu\,(2x-1)$$

valid for $Re(\nu) > 0$, $Re(\lambda) > -\tfrac{1}{2}$ and

$$f_m^{(\alpha,\beta)} \left(\begin{matrix} a_2, \ldots, a_p \\ b_3, \ldots, b_q \end{matrix} ; x \right) = \frac{(1+\alpha)_m}{m!}\ {}_{p+1}F_q \left[\begin{matrix} -m,\, m+\alpha+\beta+1,\, a_2, \ldots, a_p \\ 1+\alpha,\, \tfrac{1}{2},\, b_3, \ldots\ldots\ldots, b_q \end{matrix} ; x \right]$$

is a generalized Sister Celine's polynomial [(7), eqn. (2·2), p. 80] which reduces to Sister Celine's polynomial [(3), eqn. (1), p. 806] on putting $\alpha = \beta = 0$.

In $(4\cdot1)$ and $(4\cdot2)$, substituting $p = q = 3$, $a_2 = \rho$, $a_3 = \tfrac{1}{2}$ and $b_3 = \sigma$, we have

$(4\cdot3)$
$$\int_0^1 x^\lambda\,(1-x)^{\nu-\frac{1}{2}}\ C_n^\nu\,(2x-1)\ H_m^{(\alpha,\beta)}(\rho,\,\sigma,\,x)\ dx$$

$$= \frac{\Gamma(\nu+\tfrac{1}{2})\,\Gamma(2\nu+n)\,\Gamma(\lambda+1)\,\Gamma(\lambda-\nu+\tfrac{3}{2})\,(1+\alpha)_m}{n!\,m!\,\Gamma(2\nu)\,\Gamma(\lambda-\nu-n+\tfrac{3}{2})\,\Gamma(\lambda+\nu+n+\tfrac{3}{2})}$$

$$\times {}_5F_4 \left[\begin{matrix} -m,\, m+\alpha+\beta+1,\, \rho,\, \lambda+1,\, \lambda-\nu+\tfrac{3}{2} \\ 1+\alpha,\, \sigma,\, \lambda-\nu-n+\tfrac{3}{2},\, \lambda+\nu+n+\tfrac{3}{2} \end{matrix} ; 1 \right]$$

where $Re(\lambda) > -1$, $Re(\nu) > -\tfrac{1}{2}$.

$(4\cdot4)$
$$x^\lambda\ H_m^{(\alpha,\beta)}(\rho,\,\sigma,\,x)$$

$$= \frac{2^{2\nu}\,\Gamma(\nu)}{\sqrt{\pi}\ m!}\,\frac{\Gamma(\lambda+\nu+\tfrac{1}{2})\,(1+\alpha)_m}{\Gamma(\lambda+2\nu+1)} \sum_{r=0}^{\infty} \frac{(-1)^r\,(\nu+r)\,(-\lambda)_r}{(\lambda+2\nu+1)_r}$$

$$\times {}_5F_4 \left[\begin{matrix} -m,\, m+\alpha+\beta+1,\, \rho,\, \lambda+\nu+\tfrac{1}{2},\, \lambda+1 \\ 1+\alpha,\, \sigma,\, \lambda-r+1,\, \lambda+2\nu+r+1 \end{matrix} ; 1 \right] C_r^\nu\,(2x-1),$$

valid for $Re(\nu) > 0$, $Re(\lambda) > -\tfrac{1}{2}$ and

$$H_m^{(\alpha,\beta)}(\rho,\,\sigma,\,x) = \frac{(1+\alpha)_m}{m!}\ {}_3F_2 \left[\begin{matrix} -m,\, m+\alpha+\beta+1,\, \rho \\ 1+\alpha,\, \sigma \end{matrix} ; x \right] \text{is a generalized}$$

Rice's polynomial [(4), p. 158, (2·3)] which reduces to Rice's polynomial [(5), p. 108] when $\alpha = \beta = 0$.

Further setting $\rho = \sigma$, $\alpha = \beta = \mu - \frac{1}{2}$, in (4·3) and (4·4) and using the following relations [(1), p. 267] :

$$C_n^\nu (x) = (-1)^n \, C_n^\nu (-x), \quad C_n^\nu (x) = \frac{(2\nu)_n}{(\nu + \frac{1}{2})_n} \, P_n^{(\nu - \frac{1}{2}, \, \nu - \frac{1}{2})} (x),$$

we obtain a known result [(1), p. 283, (16)] :

$$\int_0^1 x^\lambda (1-x)^{\nu - \frac{1}{2}} \, C_n^\nu (1 - 2x) \, C_m^\mu (1 - 2x) \, dx$$

$$= \frac{\Gamma(2\mu + m) \, \Gamma(\nu + \frac{1}{2}) \, \Gamma(2\nu + n) \, \Gamma(\lambda + 1) \, \Gamma(\nu - \lambda + n - \frac{1}{2})}{m! \, n! \, \Gamma(2\mu) \, \Gamma(2\nu) \, \Gamma(\nu - \lambda - \frac{1}{2}) \, \Gamma(\lambda + \nu + n + \frac{3}{2})}$$

$$\times \, {}_4F_3 \left[\begin{matrix} -m, \, m + 2\mu, \, \lambda + 1, \, \lambda - \nu + \frac{3}{2} \\ \frac{1}{2} + \mu, \, \lambda - \nu - n + \frac{3}{2}, \, \lambda + \nu + \frac{3}{2} + n \end{matrix} \; ; 1 \right]$$

valid for $Re(\lambda) > -1$, $Re(\nu) > -\frac{1}{2}$.

(4·5)
$$x^\lambda \, C_m^\mu (1 - 2x)$$

$$= \frac{2^{2\nu} \, \Gamma(\nu) \, \Gamma(2\mu + m) \, \Gamma(\lambda + \nu + \frac{1}{2})}{m! \, \sqrt{\pi} \, \Gamma(2\mu) \, \Gamma(\lambda + 2\nu + 1)} \sum_{r=0}^\infty \frac{(-1)^r \, (\nu + r) \, (-\lambda)_r}{(\lambda + 2\nu + 1)_r}$$

$$\times \, {}_4F_3 \left[\begin{matrix} -m, \, m + 2\mu, \, \lambda + \nu + \frac{1}{2}, \, \lambda + 1 \\ \mu + \frac{1}{2}, \, \lambda - r + 1, \, \lambda + 2\nu + r + 1 \end{matrix} \; ; 1 \right] C_r^\nu (2x - 1)$$

valid for $Re(\nu) > 0$, $Re(\lambda) > -\frac{1}{2}$.

(b) With $p = 0$, $q = 1$, $b_1 = 1 + \alpha$, and multiplying both sides by $\frac{(1 + \alpha)_m}{m!}$, we have

(4·6)
$$\int_0^1 x^\lambda (1-x)^{\nu - \frac{1}{2}} \, C_n^\nu (2x - 1) \, L_m^{(\alpha)} (x) \, dx$$

$$= \frac{\Gamma(\nu + \frac{1}{2}) \, \Gamma(2\nu + n) \, \Gamma(\lambda + 1) \, \Gamma(\lambda - \nu + \frac{3}{2}) \, (1 + \alpha)_m}{n! \, m! \, \Gamma(2\nu) \, \Gamma(\lambda - \nu - n + \frac{3}{2}) \, \Gamma(\lambda + \nu + n + \frac{3}{2})}$$

$$\times \, {}_3F_3 \left[\begin{matrix} -m, \, \lambda + 1, \, \lambda - \nu + \frac{3}{2} \\ 1 + \alpha, \, \lambda - \nu - n + \frac{3}{2}, \, \lambda + \nu + n + \frac{3}{2} \end{matrix} \; ; 1 \right]$$

where $Re(\lambda) > -1$, $Re(\nu) > -\frac{1}{2}$.

(4·7)
$$x^\lambda \, L_m^{(\alpha)} (x)$$

$$= \frac{2^{2\nu} \, \Gamma(\nu) \, \Gamma(\lambda + \nu + \frac{1}{2}) \, (1 + \alpha)_m}{\sqrt{\pi} \, m! \, \Gamma(\lambda + 2\nu + 1)} \sum_{r=0}^\infty \frac{(-1)^r \, (\nu + r) \, (-\lambda)_r}{(\lambda + 2\nu + 1)_r}$$

$$\times \, {}_3F_3 \left[\begin{matrix} -m, \, \lambda + \nu + \frac{1}{2}, \, \lambda + 1 \\ 1 + \alpha, \, \lambda - r + 1, \, \lambda + 2\nu + r + 1 \end{matrix} \; ; 1 \right] C_r^\nu (2x - 1)$$

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where $Re(\nu) > 0$, $Re(\lambda) > -\frac{1}{2}$ and $L_n^{(\alpha)}(x)$ is a generalized Laguerre polynomial.

(B) Particular cases of (2·2) and (3·2) with $\delta = c = 2$:

(i) Substituting $p = 1$, $q = 2$, $a_1 = \gamma - \beta$, $b_1 = \gamma$, $b_2 = 1 - \beta - m$, $\mu = 1$

and multiplying both sides by $\dfrac{2^m (\beta)_m}{m\,!}$, we get

$$(4\cdot 8) \qquad \int_0^1 x^\lambda (1-x)^{\nu - \frac{1}{2}} C_n^\nu (2x-1) R_m(\beta, \gamma\,; x)\, dx$$

$$= \frac{2^m (\beta)_m \,\Gamma(\nu + \frac{1}{2})\, \Gamma(2\nu + n)\, \Gamma(\lambda + m + 1)\, \Gamma(\lambda + m - \nu + \frac{3}{2})}{m\,!\, n\,!\, \Gamma(2\nu)\, \Gamma(\lambda + m - \nu - n + \frac{3}{2})\, \Gamma(\lambda + m + \nu + n + \frac{3}{2})}$$

$$\times {}_7F_6 \left[\begin{array}{c} \triangle(2, -m),\ \gamma - \beta,\ \triangle(2, -\lambda - m + \nu + n - \frac{1}{2}),\ \triangle(2, -\lambda - m - \nu - n - \frac{1}{2}) \\ \gamma,\ 1 - \beta - m,\ \triangle(2, -\lambda - m),\ \triangle(2, -\lambda - m + \nu - \frac{1}{2}) \end{array} ;\ 1 \right]$$

where $Re(\lambda) > -1$, $Re(\nu) > -\frac{1}{2}$.

With $n = 0$, $\nu = M - \frac{1}{2}$, and $\lambda = L - 1$, (4·8) reduces to a known result [(7), p. 90] :

$$\int_0^1 x^{L-1} (1-x)^{M-1} R_m (\beta, \gamma\,; x)\, dx$$

$$= \frac{\Gamma(M)\, \Gamma(L+m)\, 2^m\, (\beta)_m}{m\,!\, \Gamma(L+M+m)} \quad {}_8F_4 \left(\begin{array}{c} \dfrac{-m}{2},\ \dfrac{-m+1}{2},\ \gamma - \beta,\ \dfrac{1-L-M-m}{2},\ \dfrac{2-L-M-m}{2} \\[2mm] \gamma,\ 1-\beta-m,\ \dfrac{1-L-m}{2},\ \dfrac{2-L-m}{2} \end{array} ;\ 1 \right)$$

valid for $Re(L) > 0$, $Re(M) > 0$.

$$(4\cdot 9) \qquad x^\lambda R_m(\beta, \gamma\,; x)$$

$$= \frac{2^{2\nu + m}\, (\beta)_m\, \Gamma(\nu)\, \Gamma(\lambda + \nu + m + \frac{1}{2})}{\sqrt{\pi}\, m\,!\, \Gamma(\lambda + 2\nu + m + 1)} \sum_{r=0}^\infty \frac{(-1)^r\, (\nu + r)\, (-\lambda - m)_r}{(\lambda + 2\nu + m + 1)_r}$$

$$\times {}_7F_6 \left[\begin{array}{c} \triangle(2, -m),\ \gamma - \beta,\ \triangle(2, -\lambda - m + r),\ \triangle(2, -\lambda - 2\nu - m - r) \\ \gamma,\ 1 - \beta - m,\ \triangle(2, -\lambda - \nu - m + \frac{1}{2}),\ \triangle(2, -\lambda - m) \end{array} ;\ 1 \right] C_r^\nu (2x-1)$$

valid for $Re(\nu) > 0$, $Re(\lambda) > -\frac{1}{2}$ and $R_m(\beta, \gamma\,; x)$ is a Bedient's polynomial [(6), p. 297, (1)] which reduces to the Gegenbauer polynomial $C_m^{(\beta)}(x)$ when $\underset{\gamma \to \infty}{\mathrm{Lim}}\ R_m(\beta, \gamma\,; x)$.

(ii) Setting $p = q = 0$, $\mu = -1$ and multiply both sides by 2^m, we obtain

$$(4\cdot 10) \qquad \int_0^1 x^\lambda (1-x)^{\nu - \frac{1}{2}} C_n^\nu (2x-1) H_m(x)\, dx$$

$$= \frac{2^m \Gamma(\nu + \frac{1}{2}) \, \Gamma(2\nu + n) \, \Gamma(\lambda + m + 1) \, \Gamma(\lambda + m - \nu + \frac{3}{2})}{n! \, \Gamma(2\nu) \, \Gamma(\lambda + m - \nu - n + \frac{3}{2}) \, \Gamma(\lambda + m + \nu + n + \frac{3}{2})}$$

$$\times \; {}_6F_4 \left[\begin{array}{c} \triangle(2, -m), \; \triangle(2, -\lambda - m + \nu + n - \frac{1}{2}), \; \triangle(2, -\lambda - m - \nu - n - \frac{1}{2}) \\ \triangle(2, -\lambda - m), \; \triangle(2, -\lambda - m + \nu - \frac{1}{2}) \end{array} ; -1 \right]$$

where $Re(\lambda) > -1$, $Re(\nu) > -\frac{1}{2}$.

(4·11)
$$x^\lambda \, H_m(x)$$

$$= \frac{2^{2\nu + m} \, \Gamma(\nu) \, \Gamma(\lambda + \nu + m + \frac{1}{2})}{\sqrt{\pi} \, \Gamma(\lambda + 2\nu + m + 1)} \sum_{r=0}^{\infty} \frac{(-1)^r \, (\nu + r) \, (-\lambda - m)_r}{(\lambda + 2\nu + m + 1)_r}$$

$$\times \; {}_6\dot{F}_4 \left[\begin{array}{c} \triangle(2, -m), \; \triangle(2, -\lambda - m + r), \; \triangle(2, -\lambda - 2\nu - m - r) \\ \triangle(2, -\lambda - \nu - m + \frac{1}{2}), \; \triangle(2, -\lambda - m) \end{array} ; -1 \right] C_r^\nu (2x - 1)$$

valid for $Re(\nu) > 0$, $Re(\lambda) > -\frac{1}{2}$ and $H_m(x)$ is the Hermite polynomial.

(iii) With $p = 0$, $q = 1$, $b_1 = \frac{1}{2} - m$, $\mu = 1$ and mutiplying both sides by $\frac{2^m (\frac{1}{2})_m}{m!}$, we have

(4·12)
$$\int_0^1 x^\lambda \, (1-x)^{\nu - \frac{1}{2}} \; C_n^\nu (2x - 1) \, P_m(x) \, dx$$

$$= \frac{2^m (\frac{1}{2})_m \, \Gamma(\nu + \frac{1}{2}) \, \Gamma(2\nu + n) \, \Gamma(\lambda + m + 1) \, \Gamma(\lambda + m - \nu + \frac{3}{2})}{m! \, n! \, \Gamma(2\nu) \, \Gamma(\lambda + m - \nu - n + \frac{3}{2}) \, \Gamma(\lambda + m + \nu + n + \frac{3}{2})}$$

$$\times \; {}_6F_5 \left[\begin{array}{c} \Delta(2, -m), \; \Delta(2, -\lambda - m + \nu + n - \frac{1}{2}), \; \Delta(2, -\lambda - m - \nu - n - \frac{1}{2}) \\ \frac{1}{2} - m, \; \Delta(2, -\lambda - m), \; \Delta(2, -\lambda - m + \nu - \frac{1}{2}) \end{array} ; 1 \right]$$

where $Re(\lambda) > -1$, $Re(\nu) > -\frac{1}{2}$.

(4·13)
$$x^\lambda \, P_m(x)$$

$$= \frac{2^{2\nu + m} \, \Gamma(\nu) \, \Gamma(\lambda + \nu + m + \frac{1}{2}) \, (\frac{1}{2})_m}{\sqrt{\pi} \, m! \, \Gamma(\lambda + 2\nu + m + 1)} \cdot \sum_{r=0}^{\infty} \frac{(-1)^r \, (\nu + r) \, (-\lambda - m)_r}{(\lambda + 2\nu + m + 1)_r}$$

$$\times \; {}_6F_5 \left[\begin{array}{c} \triangle(2, -m), \; \triangle(2, -\lambda - m + r), \; \triangle(2, -\lambda - 2\nu - m - r) \\ \frac{1}{2} - m, \; \triangle(2, -\lambda - \nu - m + \frac{1}{2}), \; \triangle(2, -\lambda - m) \end{array} ; 1 \right] C_r^\nu (2x - 1)$$

where $Re(\nu) > 0$, $Re(\lambda) > -\frac{1}{2}$ and $P_m(x)$ is the Legendre polynomial.

(c) Special case :

With $\mu = 0$, $\delta = 1$ either in (3·1) or (3·2) and replacing x by $\left(\frac{1 - X}{2} \right)$ and

using the relation

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$$\dot{C}_n^\nu (\dot{X}) = (-1)^n \ \dot{C}_n^\nu (-\ddot{X}),$$

we obtain a known result [(2), p. 213, (6)] :

$$(1 - X)^\lambda = 2^{2\nu+\lambda} \ \pi^{-\frac{1}{2}} \ \Gamma(\nu) \ \Gamma(\lambda+\nu+\tfrac{1}{2}) \sum_{r=0}^{\infty} \frac{(\nu+r)(-\lambda)_r}{\Gamma(\lambda+2\nu+r+1)} \ C_r^\nu (X),$$

$-1 < X < 1, -\lambda < \tfrac{1}{2}(\nu+1)$ if $\nu \geqslant 0, -\lambda < \tfrac{1}{2}+\nu$ if $-\tfrac{1}{2} < \nu \leqslant 0$.

Acknowledgement

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On some results involving H-functions and associated Legendre Functions

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Abstract

In this paper the integral involving H-and associated Legendre functions has evaluated. This integral has been employed to establish the expansion formula for the H–function in series of the associated Legendre functions.

1. Fox [(5), p. 408] has recently introduced the H–function in the form of Mellin-Barnes type integral as

(1·1)
$$H^{m,\,n}_{p,\,q}\left[x \;\middle|\; \begin{matrix} (a_1, a_1), (a_2, \alpha_2), \ldots\ldots, (a_p, a_p) \\ (b_1, \beta_1), (b_2, \beta_2), \ldots\ldots, (b_q, \beta_q) \end{matrix} \right]$$

$$= \frac{1}{2\pi i} \int_L \frac{\prod\limits_{j=1}^{m} \Gamma(b_j - \beta_j\xi) \; \prod\limits_{j=1}^{n} \Gamma(1 - a_j + a_j\xi)}{\prod\limits_{j=m+1}^{q} \Gamma(1 - b_j + \beta_j\xi) \prod\limits_{j=n+1}^{p} \Gamma(a_j - a_j\xi)} \; x^{\xi} \; d\xi$$

where x is not equal to zero and an empty product is interpreted as unity; p, q, m, n are integers satisfying $1 \leqslant m \leqslant q,\ 0 \leqslant n \leqslant p$; $a_j(j = 1, \ldots\ldots, p)$, $\beta_j(j = 1, \ldots, q)$ are positive numbers and $a_j(j = 1, \ldots, p)$, $b_j(j = 1, \ldots, q)$ are complex numbers such that no pole of $\Gamma(b_h - \beta_h\xi)$, $(h = 1, \ldots, m)$ coincides with any pole of $\Gamma(1 - a_i + \alpha_i\xi)$, $(i = 1, \ldots, n)$ i.e.,

(1·2)
$$\alpha_i(b_h + r) \neq (a_i - \eta - 1)\, \beta_h$$

$$(\gamma, \eta = 0, 1, \ldots, ; h = 1, \ldots, m ; i = 1, \ldots, n).$$

Further the contour L runs from $\sigma - i\infty$ to $\sigma + i\infty$ such that the points :

(1·3)
$$\xi = \frac{(b_h + \gamma)}{\beta_h}, (h = 1, \ldots, m : \gamma = 0, 1, \ldots.)$$

which are poles of $\Gamma(b_h - \beta_h\xi)$ lie to the right and the points :

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$$(1\cdot4) \qquad \xi = \frac{(a_i - \eta - 1)}{a_i} \, , (i = 1, \ldots, n, \eta = 0, 1, \ldots \ldots)$$

which are poles of $\Gamma(1 - a_i + a_i\xi)$ lie to the left of L. Such a contour is possible on account of $(1\cdot2)$. These assumptions for the H-function will be adhered to throughout this paper.

Braaksma [(1), p. 278] has studied in detail the asymptotic expansions and analytic continuations for a class of Barnes-integrals.

The associated Legendre function has defined by MacRobert [(6), p. 123]:

$$(1\cdot5) \qquad P_n^m(x) = \frac{\Gamma(n+m+1) \, (-1)^{\frac{1}{2}m}}{\Gamma(n-m+1) \, m!} \cdot \left(\frac{1-x}{1+x} \right)^{\frac{1}{2}m} F[-n, \, n+1; \, m+1; \, \tfrac{1}{2}(1-x) \,]:$$

where m is a positive integer and n is unrestricted.

We shall abbreviate the H-function $(1\cdot1)$ as

$$(1\cdot6) \qquad H \begin{array}{c} m, \, n \\ p, \, q \end{array} \left[\, x \, \Big| \, \begin{array}{c} \{ (a_p, \, \alpha_p) \} \\ \{ (b_q, \, \beta_q) \} \end{array} \, \right]$$

where $\{ (a_p, \, \alpha_p) \}$ represents the set of parameters $(a_1, \, a_1), \ldots \ldots \ldots, (a_p, \, a_p)$ and similarly for $\{ (b_q, \, \beta_q) \}$.

The symbol $\triangle (m, \, n)$ stands for the parameters :

$$\frac{n}{m}, \, \frac{n+1}{m}, \, \ldots \ldots, \, \frac{n+m-1}{m} \, .$$

$\mathbf{2}\cdot$ In this section, we have derived the following integral.

The formula to be proved is

$$(2\cdot1) \qquad \int_0^1 x^{\frac{1}{2}\mu+k} \, (1-x)^{\frac{1}{2}\mu} \, P_l^{\mu} \, (2x-1) \, H \begin{array}{c} m, \, n \\ p, \, q \end{array} \left[\, zx^\delta \, \Big| \, \begin{array}{c} \{ (a_p, \, \alpha_p) \} \\ \{ (b_q, \, \beta_q) \} \end{array} \, \right] dx$$

$$= \frac{(-1)^{\frac{1}{2}\mu} \, \Gamma(\mu+l+1)}{\mu! \, \Gamma(l-\mu+1) \, \delta^{\mu+1}} \, H \begin{array}{c} m,n+2\delta \\ p+2\delta,q+2\delta \end{array} \left[(z) \, \Big| \, \begin{array}{c} (\triangle(\delta,-k),1), \, (\triangle(\delta,-k-\mu),1), \, \{(a_p,\alpha_p)\} \\ \{(b_q,\beta_q)\},(\triangle(\delta,-k-\mu-l-1),1),(\triangle(\delta,-k-\mu+2),1) \end{array} \right]$$

where δ, μ are positive integers, and l is unrestricted, $k > -\mu - 1$, and

$$\sum_1^p \alpha_j - \sum_1^q \beta_j \equiv \mathcal{T} \leqslant 0, \quad \sum_1^n \alpha_j - \sum_{n+1}^p \alpha_j + \sum_1^m \beta_j - \sum_{m+1}^q \beta_j \equiv \lambda > 0, \, |\arg z| < \tfrac{1}{2}\pi\lambda$$

and $Re\left(1 + \delta \, \frac{b_h}{\beta_h} \right) > 0, \, (h = 1, \ldots \ldots, m).$

[504]

Proof :

To prove (2·1), we express the H–function in the integrand of (2·1) in the form of Mellin–Barnes type of integral (1·1) and change the order of integration which is easily seen to be justified under the condition stated in (2·1), we obtain

$$(2\cdot2) \quad \frac{1}{2\pi i}\int \frac{\prod\limits_{j=1}^{m}\Gamma(b_j-\beta_j s)\prod\limits_{j=1}^{n}\Gamma(1-a_j+\alpha_j s)\; z^s}{\prod\limits_{j=m+1}^{q}\Gamma(1-b_j+\beta_j s)\prod\limits_{j=n+1}^{p}\Gamma(a_j-\alpha_j s)}\left\{\int_0^1 x^{\frac{1}{2}\mu+k+\delta s}(1-x)^{\frac{1}{2}\mu}\;P_l^{\mu}(2x-1)dx\right\}ds$$

Now evaluating x–integral with the help of the known result [(2), p. 104,(2·2)]:

$$\int_0^1 x^{\frac{1}{2}m+p}(1-x)^{\frac{1}{2}m}\;P_n^{m}(2x-1)\;dx = \frac{(-1)^{\frac{1}{2}m}\;\Gamma(m+n+1)\;\Gamma(p+1)\;\Gamma(p+m+1)}{m!\;\Gamma(n-m+1)\;\Gamma(p+m+n+2)\;\Gamma(p+m-n+1)}$$

where m is a positive integer and $p > -m-1$, and using the Gauss' multiplication theorem for Gamma functions [(3), p. 4, (11)] :

$$\Gamma(mz) = (2\pi)^{\frac{1}{2}(1-m)}\;m^{mz-\frac{1}{2}}\prod\limits_{l=1}^{m}\Gamma\left(z+\frac{l-1}{m}\right)$$

where m is a positive integer, (2·2) reduces to

$$(2\cdot3) \qquad \frac{(-1)^{\frac{1}{2}\mu}\;\Gamma(\mu+l+1)}{\mu!\;\Gamma(l-\mu+1)\;\delta^{\mu+1}}$$

$$\times\frac{1}{2\pi i}\int_L \frac{\prod\limits_{j=1}^{m}\Gamma(b_j-\beta_j s)\prod\limits_{j=1}^{n}\Gamma(1-a_j+\alpha_j s)\prod\limits_{i=0}^{\delta-1}\Gamma\left(\frac{k+1+i}{\delta}+s\right)}{\prod\limits_{j=m+1}^{q}\Gamma(1-b_j+\beta_j s)\prod\limits_{j=n+1}^{p}\Gamma(a_j-\alpha_j s)\prod\limits_{i=0}^{\delta-1}\Gamma\left(\frac{k+\mu+l+2+i}{\delta}+s\right)}\times$$

$$\frac{\prod\limits_{i=0}^{\delta-1}\Gamma\left(\frac{k+\mu+1+i}{\delta}+s\right)z^s}{\prod\limits_{i=0}^{\delta-1}\left(\frac{k+\mu-l+1+i}{\delta}+s\right)}\;ds$$

which yields the value of the integral (2·1) in accordance with the definition of the H–function (1·1).

3. Expansion

The expansion formula to be established is

$$(3\cdot1) \qquad x^{\frac{1}{2}\mu+k}(1-x)^{\frac{1}{2}\mu}\,H_{p,q}^{m,n}\!\left[zx^{\delta}\;\middle|\;\begin{array}{c}\{(a_p,\alpha_p)\}\\\{(b_q,\beta_q)\}\end{array}\right]$$

$$= \frac{(-1)^{\frac{1}{2}\mu}}{\mu!\,\delta^{\mu+1}} \sum_{r=0}^{\infty} (2r+1)\, H^{m,n+2\delta}_{p+2\delta,q+2\delta}$$

$$\times \left[z \left| \begin{matrix} (\triangle(\delta,-k),1),\ (\triangle(\delta,-k-\mu),1),\ \{(a_p,\alpha_p)\} \\ \{(b_q,\beta_q)\},\ (\triangle(\delta,-\mu-k-r-1),1),\ (\triangle(\delta,-k-\mu+r),1) \end{matrix} \right. \right] P_r^{\mu}(2x-1)$$

where μ, δ are positive integers, $k > -\mu - 1$ and

$$\sum_1^p \alpha_j - \sum_1^q \beta_j \equiv \mathcal{J} \leqslant 0,\quad \sum_1^n \alpha_j - \sum_{n+1}^p \alpha_j + \sum_1^m \beta_j - \sum_{m+1}^q \beta_j \equiv \lambda > 0,\quad |\arg z| < \tfrac{1}{2}\pi\lambda$$

and $Re\left(1 + \delta \dfrac{b_h}{\beta_h}\right) > 0,\ (h = 1, \ldots, , m).$

Proof: Let

$$(3\cdot2) \qquad f(x) = x^{\frac{1}{2}\mu+k}(1-x)^{\frac{1}{2}\mu}\, H^{m,n}_{p,q}\left[zx^{\delta} \left| \begin{matrix} \{(a_p,\alpha_p)\} \\ \{(b_q,\beta_q)\} \end{matrix} \right. \right] = \sum_{r=0}^{\infty} C_r\, P_r^{\mu}(2x-1),$$

$$(0 < x < 1).$$

Equation $(3\cdot2)$ is valid since $f(x)$ is continuous and of bounded variation in the open interval $(0, 1)$.

Now multiply both sides of $(3\cdot2)$ by $P_l^{\mu}(2x-1)$ and integrate with respect to x from 0 to 1. Change the order of integration and summation (which is permissible) on the right, we have

$$(3\cdot3) \qquad \int_0^1 x^{\frac{1}{2}\mu+k}(1-x)^{\frac{1}{2}\mu}\, P_l^{\mu}(2x-1)\, H^{m,n}_{p,q}\left[zx^{\delta} \left| \begin{matrix} \{(a_p,\alpha_p)\} \\ \{(b_q,\beta_q)\} \end{matrix} \right. \right] dx$$

$$= \sum_{r=0}^{\infty} C_r \int_0^1 P_l^{\mu}(2x-1)\, P_r^{\mu}(2x-1)\, dx.$$

Using the orthogonality property for the associated Legendre functions [(4), p. 279, (27)] :

$$\int_0^1 \left[P_n^m(2x-1) \right]^2 dx = \frac{1}{(2n+1)} \cdot \frac{(n+m)!}{(n-m)!},\ m \leqslant n$$

on the right and the result $(2\cdot1)$ on the left of $(3\cdot3)$, we obtain

$$(3\cdot4) \qquad C_l = \frac{(-1)^{\frac{1}{2}\mu}\,(2l+1)}{\mu!\,\delta^{\mu+1}}$$

$$\times\, H^{m,n+2\delta}_{p+2\delta,q+2\delta}\left[z \left| \begin{matrix} (\triangle(\delta,-k),1),\ (\triangle(\delta,-k-\mu),1),\ \{(a_p,\alpha_p)\} \\ \{(b_q,\beta_q)\},\ (\triangle(\delta,-k-\mu-l-1),1),\ (\triangle(\delta,-k-\mu+l),1) \end{matrix} \right. \right].$$

With the help of $(3\cdot2)$ and $(3\cdot4)$, the expansion formula $(3\cdot1)$ is obtained.

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The H-function is in a more generalized form which yields many know functions on specializing the parameters. By taking $\alpha_j = \beta_h = 1$ $(i = 1, \ldots, p;$ $h = 1, \ldots, q)$ in $(1\cdot1)$, the H-function reduces to Meijer's G-functions $[\,(3), \text{ p. } 207, (1)\,]$ which is itself a more generalized function of many known special functions used in pure and applied branches of Mathematics $[(3), \text{ p. } 215\text{--}222]$.

Hence the formulae established in this paper are of general character.

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Changes in the Physico-chemical properties of a soil on the addition of weeds as a source of organic matter

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Abstract

Incubation studies were made to find the changes in the physico-chemical properties of a soil as a result of humification of weeds both under arable as well as under submerged conditions. It was reported earlier that the addition of weeds to this soil caused significant increase in the yield of wheat and paddy grain and straw.

In general, an initial decrease in pH and increase in conductivity was observed under both the conditions. Subsequently, pH increased and conductivity decreased. These changes were within the limits specified for the healthy growth of crop plants. The total exchangeable bases, the exchangeable calcium, magnesium, potassium and sodium increased. The increase was more under submerged conditions than under well-drained conditions. The humic acid content also improved. Under well drained conditions an increase in percentage water-stable aggregates, maximum water holding capacity and sticky point was noted.

Introduction

It was reported that the addition of weeds to a soil caused significant increase in the yield of paddy grain and straw[19]. Similarly, incorporation of weeds increased the yield of wheat crop under the arable conditions[22]. Incubation studies made separately, revealed that available NPK increased appreciably as a result of humification of weeds in the soil. Since, the growth is related to soil physical conditions also, the present investigations were undertaken to determine the changes in the physico-chemical properties of this soil *e.g.* pH, conductivity, the exchangeable bases, humic acid content, percentage water-stable aggregates etc. which take place on the addition of weeds under arable as well submerged conditions.

Experimental

(*i*) *Materials* : Surface soil, 022·5 cms. was collected from the main block of the Agronomy Division Farm, I. A. R. I. Delhi. The soil was air dried and passed through 2 mm. sieve for incubation studies and chemical analysis. The physico-chemical properties of the soil are given in Table 1.

TABLE 1

Physico-chemical properties of the soil

Conductivity (mhos/cm at 25°C)	0·70
pH (1 : 2·5)	8·05
Maximum water-holding capacity	33·81%
Sticky point	10·09%

Texture class	Sandy—Clay loam
Water-stable aggregates :	
2 mm	0·76%
2–1 mm	3·84%
1–0·5 mm	19·12%
0·5–0·25 mm	11·66%
Total water-stable aggregates/0·25 mm	35·38%
Organic carbon	0·561%
Total Nitrogen	0·065%
C/N ratio	8·62
Exchangeable bases :	
Calcium	11·14 (meq/100 gm soil)
Magnesium	2·21 ,,
Potassium	0·90 ,,
Sodium	0·32 ,,
CEC	15·52 ,,

Twentyone weeds commonly available in kharif and rabi seasons were collected at the flowering stage from the Agronomy Division Farm, I. A. R. I. The weeds were identified for their botanical names. These were analysed for organic and inorganic constituents. The results of analysis have been published.[20] However, it is pertinent to record that these were found quite rich in minerals like N, P, K, Na, Ca, Mg, S, etc. The plant materials were screened after grinding through 2 mm sieve for incubation experiments.

(ii) *Incubation studies* : 800 gms. of 2 mm sieved soil was mixed with a screened plant material and kept in a wide mouthed glass jar of 2 lbs. capacity each at 35°C. The different weeds were added @ 2% soil weight. The experiments were conducted in two series, one under well-drained and the other under submerged conditions. In the first series, moisture was maintained at 50% of maximum water-holding capacity. In the second series, submerged conditions were maintained with 2·5 cms. free film of water during the entire period. The mouths of the jar were tightened with perforated polythene sheets so as to check excessive moisture loss by evaporation but, at the same time ensure adequate aeration of the soil. Incubation period lasted from 1·6·66 to 28·9·66. Soil samples were drawn from both the series at periodical intervals of 10, 20, 40, 80 and 120 days to determine various physico-chemical properties.

(iii) *Soil analysis* : pH was determined using Beckman glass electrode pH meter in 1 : 2·5 soil water suspension. Conductivity was measured with the help of a solubridge in 1 : 2·5 filtered water extract and expressed in millimhos/cm at 25°C. Keen-Rackzowski boxes were used for the estimation of water-holding capacity as outlined in Piper[15]. Different fractions of water-stable aggregates were determined according to Yoder[23] : sticky point by a procedure due to Coutts[5]. Exchangeable bases were estimated using Puri's method[17] and total cation exchange capacity by neutral ammonium acetate leaching method of Schollenberger as mentioned in Piper[15]. Humic acid content was found by Chaminade's method[3] with some modifications discussed by Vimal[22]. The periods effect represented in some of the tables are the average mean values.

Results and Discussions

(a) Changes in soil reaction (pH) during humification of weeds both under well-drained and submerged conditions :

It is evident from table 2 (a and b) that on the addition of weeds, there was a decrease in pH upto first ten days both under well-drained and submerged conditions. Subsequently, pH increased and did not suffer marked changes. In the initial stages, during the decomposition of organic matter, the formation of carbon dioxide and organic acids cause a decrease in soil pH. Subseqent increase in pH may be attributed to the consumption of organic acids by micro-organisms, decrease of CO_2, accumulation of NH_3 as affected by protein content of organic materials and release of exchangeable bases e.g. Ca, Mg, Na etc.

In the experiments made pH varied from 6·8 to 7·8 as found at different periods. This pH range is favourable for the healthy growth of crop plants. According to International Rice Research Institute[8], stabilization of pH in soils from 6·5 to 7·5, 2–3 weeks following flooding has an important bearing on the nutrient uptake by rice plants. A pH of 6·5–7·5 increases the rate of organic matter mineralisation (Mitsui[10]), enhances the destruction of organic acids (Acharya[1]), decreases the concentration of CO_2, Fe^{++} and Al^{+++} (Ponnamperuma[16]), favours denitrification (De and Sarkar[6] ; Delwiche[7]) and reduces phosphate fixation (Aoki[2]).

(b) Changes in specific conductivity during humification of weeds both under well-drained and submerged conditions :

Data in table 3 (a and b) showed that under all the treatments, conductivity at first increased and thereafter it decreased. The changes in conductivity as of pH are related to the liberation, accumulation and loss of the products of plant and microbial origin. Sharma and Bhattacharya[18] found that the addition of organic matter at first resulted in increased conductivity and, decreased soil pH.

A comparison of the table (a) with (b) revealed that on submergence, there was a greater increase in specific conductivity as compared to well-drained conditions. This is compatible with more exchangeable cations, Ca, Mg, Na and K found under anaerobic conditions. It is interesting to observe that specific conductivity never exceeded 4 millimhos/cm which is the limit specified by the United States Salinity Laboratory[21] for the healthy growth of plants.

(c) Exchangeable calcium, magnesium, sodium, potassium and total bases during humification of weeds both under well-drained and submerged conditions :

It is seen from the table 4 (a and b) that on the addition of weeds, the content of exchangeable bases increased. Calcium, magnesium, sodium potassium and total exchangeable base varied from 9·19–12·73, 1·77–3·42, 0·18–0·93, 0 61–1·97 and 11·76–20·94 meq/100 gm. soil, and from 10·51–15·03, 1·90–3·97, 0·27–1·25, 0·76 5 92 and 13·44–24·62 meq/100 gm under well-drained and submerged conditions respectively. This increase in exchangeable bases is related to the observed increase in humic acid content (table 5) and conductivity (table 3). Mc-George[12] found that lignin, ligno-hemicelluloses and ligno-cellulose fractions function largely as exchangeable compounds of soil organic matter. Xylan, a constituent of hemicelluloses is particularly important. An increase in base exchange capacity due to the decomposition of organic matter has been reported by Peevy and Norman[14].

TABLE 2(a)

Changes in soil reaction (pH) during humification of weeds under well-drained conditions

Days / Treatment	0	10	20	40	80	120
C. Soil alone	8·05	7·95	8·00	7·95	8·00	8·00
1 Soil + T. monogyna	7·75	7·60	7·80	7·95	7·85	7·75
2 Soil + L. Camara	7·65	7·35	7·65	7·75	7·70	7·65
3 Soil + H. eichwaldii	7·70	7·45	7·60	7·85	7·75	7·70
4 Soil + C. sparsiflorus	7·35	7·05	7·35	7·55	7·50	7·45
5 Soil + C. sativa	7·40	7·15	7·40	7·50	7·50	7·45
6 Soil + X-strumarium	7·65	7·35	7·45	7·65	7·70	7·60
7 Soil + C. oxyacantha	7·70	7·30	7·35	7·45	7·60	7·60
8 Soil + C. murale	7·75	7·45	7·55	7·65	7·75	7·70
9 Soil + F. parviflora	7·40	7·05	7·40	7·50	7·50	7·45
10 Soil + A. tenuifolius	7·35	7·10	7·35	7·45	7·50	7·40
11 Soil + A. arvensis	7·45	7·20	7·40	7·55	7·60	7·55
12 Soil + S. arvensis	7·65	7·40	7·50	7·60	7·65	7·65
13 Soil + C. album	7·75	7·45	7·60	7·65	7·70	7·75
14 Soil + M. indica	7·65	7·25	7·45	7·60	7·70	7·65
15 Soil + M. denticulata	7·45	7·15	7·45	7·50	7·55	7·50
16 Soil + C. didymus	7·55	7·10	7·35	7·45	7·45	7·45
17 Soil + P. lanceolata	7·65	7·55	7·70	7·80	7·75	7·65
18 Soil + P. minor	7·30	7·05	7·25	7·35	7·45	7·45
19 Soil + C. dactylon	7·35	7·10	7·35	7·40	7·50	7·40
20 Soil + C. rotundus	7·40	7·10	7·30	7·35	7·45	7·45
21 Soil + A. viridus	7·65	7·35	7·50	7·55	7·70	7 65
Periods effect	7·57	7·29	7·49	7·59	7·63	7·58

TABLE 2(b)

Changes in soil reaction (pH) during humification of weeds under submerged conditions

Treatment	0	10	20	40	80	120
C. Soil alone	8·05	7·75	7·90	7·95	8·00	8·00
1 Soil + T. monogyna	7·75	7·25	7·50	7·55	7·65	7·85
2 ,, + L. camara	7·65	7·15	7·40	7·50	7·65	7·70
3 ,, + H. eichwaldii	7·70	7·10	7·30	7·45	7·60	7·75
4 ,, + C. sparsiflorus	7·35	6·90	7·15	7·30	7·45	7·55
5 ,, + C. sativa	7·40	6·95	7·35	7·40	7·60	7·70
6 ,, + X. strumarium	7·65	6·90	7·30	7·40	7·65	7·70
7 ,, + C. oxyacantha	7·70	7·05	7·35	7·60	7·70	7·80
8 ,, + C. murale	7·75	7·25	7·45	7·60	7·75	7·80
9 ,, + F. parviflora	7·40	6·85	7·10	7·30	7·55	7·65
10 ,, + A. tenuifolius	7·35	6·80	7·05	7·20	7·45	7·50
11 ,, + A. arvensis	7·45	6·85	7·00	7·30	7·40	7·45
12 ,, + S. arvensis	7·65	7·25	7·40	7·65	7·70	7·75
13 ,, + C. album	7·75	7·25	7·50	7·60	7·75	7·85
14 ,, + M. indica	7·65	7·05	7·25	7·55	7·65	7·70
15 ,, + M. denticulata	7·45	7·10	7·35	7·55	7·60	7·65
16 ,, + C. didymus	7·55	6·85	7·10	7·25	7·50	7·50
17 ,, + P. lanceolata	7·65	7·20	7·45	7·55	7·65	7·75
18 ,, + P. minor	7·30	6·70	6·90	7·25	7·40	7·50
19 ,, + C. dactylon	7·35	6·85	7·00	7·15	7·35	7·45
20 ,, + C. rotundus	7·40	6·80	7·10	7·20	7·40	7·50
21 ,, + A. viridus	7·65	7·15	7·35	7·60	7·70	7·75
Period effect	7·57	7·04	7·28	7·45	7·59	7·67

TABLE 3(a)

Changes in specific conductivity during humification of weeds under well-drained conditions

Days Treatments	0	10	20	40	80	120
			(Millimhos/cm at 25°C)			
C. Soil alone	0·70	0·80	1·00	0·95	1·10	1·00
1 Soil + T. monogyna	1·65	2·15	2·90	3·10	2·60	2·40
2 ,, + L. camara	1·50	2·10	3·00	2·75	2·60	2·30
3 ,, + H. eichwaldii	1·35	2·00	2·55	2·85	2·90	2·25
4 ,, + C. sparsiflorus	1·20	1·50	1·85	2·65	2·75	2·00
5 ,, + C. sativa	1·35	1·80	2·10	2·80	3·05	1·85
6 ,, + X. strumarium	1·65	1·90	2·50	2·75	2·60	1·95
7 ,, + C. oxyacantha	1·50	2·10	2·45	2·80	3·00	2·45
8 ,, + C. murale	1·80	1·95	2·55	2·95	3·05	2·10
9 ,, + F. parviflora	1·35	2·05	2·40	2·80	2·65	1·80
10 ,, + A. tenuifolius	1·20	1·35	1·90	2·05	2·00	1·60
11 ,, + A. arvensis	1·30	2·10	2·50	2·85	2·65	2·15
12 ,, + S. arvensis	1·45	2·25	2·75	2·95	2·40	2·25
13 ,, + C. album	1·85	2·20	3·25	2·85	2·75	2·60
14 ,, + M. indica	1·40	1·90	2·25	2·75	2·40	2·10
15 ,, + M. denticulata	1·35	2·00	2·35	2·95	2·55	2·05
16 ,, + C. didymus	1·20	2·05	2·30	2·55	2·25	1·85
17 ,, + P. lanceolata	1·45	2·25	2·75	2·95	2·40	2·25
18 ,, + P. minor	1·10	1·85	2·00	2·20	2·00	1·75
19 ,, + C. dactylon	1·25	1·60	2·20	2·20	1·90	1·65
20 ,, + C. rotundus	1·20	1·35	1·90	2·00	2·05	1·60
21 ,, + A. viridus	1·70	2·10	2·35	2·80	3·10	2·70
Periods effect	1·40	1·88	2·35	2·61	2·49	2·03

TABLE 3(b)

Changes in specific conductivity during humification of weeds under submerged conditions

Days Treatments	0	10	20	40	80	120
C. Soil alone	0·70	0·85	1·05	1·25	1·15	1·05
1 ,, + T. monogyna	1·65	2·95	3·40	3·50	2·85	2·25
2 ,, + L. camara	1·50	2·65	2·90	3·15	2·45	2·20
3 ,, + H. eichwaldii	1·35	2·25	3·50	3·00	2·80	1·95
4 ,, + C. sparsiflorus	1·20	1·95	3·10	2·65	2·60	2·25
5 ,, + C. sativa	1·35	2·45	3·25	3·00	2·80	2·10
6 ,, + X. strumarium	1·65	2·80	3·35	3·15	2·70	2·05
7 ,, + C. oxyacantha	1·50	2·85	3·10	3·25	2·60	2·20
8 ,, + C. murale	1·80	2·90	3·60	3·20	2·70	2·15
9 ,, + F. parviflora	1·35	2·70	2·90	3·10	2·80	2·00
10 ,, + A. tenuifolius	1·20	2·05	2·35	2·15	2·05	1·95
11 ,, + A. arvensis	1·30	1·90	2·75	3·30	2·90	1·80
12 ,, + S. arvensis	1·45	2·55	3·25	3·00	2·80	2·10
13 ,, + C. album	1·85	3·00	3·50	2·85	2·25	1·95
14 ,, + M. indica	1·40	2·00	2·70	3·20	2·10	1·90
15 ,, + M. denticulata	1·35	2·40	3·15	2·65	2·45	1·95
16 ,, + C. didymus	1·30	2·25	3·00	2·90	2·20	1·65
17 ,, + P. lanceolata	1·45	2·75	3·50	2·95	2·30	2·05
18 ,, + P. minor	1·10	1·80	2·20	2·35	2·15	1·60
19 ,, + C. dactylon	1·25	1·90	2·45	2·25	2·10	1·80
20 ,, + C. rotundus	1·20	1·85	2·25	2·35	1·95	1·85
21 ,, + A. viridus	1·70	2·40	2·95	3·45	2·60	2·25
Periods effect	1·40	2·33	2·92	2·84	2·42	1·96

TABLE 4(a)

Effect of weeds on exchangeable Ca, Mg, Na, K and total bases at the end of 120 days period under well-drained conditions

Exchangeable bases / Treatments	Ca	Mg	Na	K	Total exchangeable bases
	(Meq./100g Soil)				
C. Soil alone	9·19	1·77	0·18	0·61	11·75
1 ,, + T. monogyna	11·84	2·74	0·89	2·14	17·61
2 ,, + L. camara	12·17	2·32	0·64	1·54	16·67
3 ,, + H. eichwaldii	11·64	2·21	0·43	1·96	16·24
4 ,, + C. sparsiflorus	11·41	2·37	0·38	1·53	15·59
5 ,, + C. sativa	11·62	2·52	0·33	1·81	16·28
6 ,, + X. strumarium	11·96	2·71	0·42	2·73	17·82
7 ,, + C. oxyacantha	12·07	2·29	0·51	1·82	16·63
8 ,, + C. murale	12·14	2·81	0·61	4·12	19·68
9 ,, + F. parviflora	11·64	2·28	0·48	1·54	19·94
10 ,, + A. tenuifolius	11·38	1·81	0·59	1·12	14·90
11 ,, + A. arvensis	11·46	2·02	0·35	1·81	15·64
12 ,, + S. arvensis	10·17	3·42	0·38	2·07	15·98
13 ,, + C. album	12·42	2·74	0·79	4·97	20·49
14 ,, + M. indica	11·53	2·48	0·51	1·92	16·44
15 ,, + M. denticulata	11·58	2·69	0·48	1·41	16·16
16 ,, + C. didymus	11·60	2·32	0·41	2·07	16·40
17 ,, + P. lanceolata	12·73	2·39	0·93	1·81	17·86
18 ,, + P. minor	10·69	1·96	0·47	1·32	14·44
19 ,, + C. dactylon	11·36	2·28	0·43	1·56	15·63
20 ,, + C. rotundus	11·32	2·42	0·28	1·22	15·24
21 ,, + A. viridus	11·90	2·86	0·38	2·54	17·58

TABLE 4(b)

Effect of weeds on exchangeable Ca, Mg, Na, K and total bases at the end of 120 days period under submerged conditions

Treatments	Ca	Mg	Na	K	Total exchangeable bases
C. Soil alone	10·51	1·90	0·27	0·76	13·44
1 Soil + T. monogyna	13·97	3·02	1·25	2·91	21·13
2 ,, + L. camara	14·37	2·93	0·86	1·96	20·12
3 ,, + H. eichwaldii	13·92	2·72	0·71	2·67	20·02
4 ,, + C. sparsiflorus	13·53	2·96	2·83	2·39	19·71
5 ,, + C. sativa	13·79	3·18	0·78	2·71	20·46
6 ,, + X. strumarium	14·40	3·06	0·81	3·15	21·42
7 ,, + C. oxyacantha	14·15	2·86	0·79	2·10	19·90
8 ,, + C. murale	14·53	3·22	0·84	5·36	23·95
9 ,, + F. parviflora	13·96	3·13	0·73	2·78	20·60
10 ,, + A. tenuifolius	13·58	2·19	0·76	1·98	18·51
11 ,, + A. arvensis	13·84	2·27	0·62	2·18	18·93
12 ,, + S. arvensis	13·12	3·97	0·68	3·84	21·61
13 ,, + C. album	14·58	3·19	0·93	5·92	24·62
14 ,, + M. indica	13·72	3·20	0·82	2·86	20·60
15 ,, + M. denticulata	13·89	3·04	0·71	1·98	19·62
16 ,, + C. didymus	13·85	2·61	0·73	2·69	19·88
17 ,, + P. lanceolata	15·03	2·73	1·10	2·14	21·00
18 ,, + P. minor	13·09	2·28	0·75	1·67	17·79
19 ,, + C. dactylon	13·46	2·86	0·69	1·76	18·77
20 ,, + C. rotundus	13·51	2·74	0·72	2·08	19·05
21 ,, + A. viridus	14·33	3·54	0·56	3·10	21 47

TABLE 5

Effect of weeds on humic acid content at the end of 120 days period both under well-drained and submerged conditions

Treatments	Well-drained	Submerged
	(Percent)	
C. Soil alone	0·044	0·047
1 Soil + T. monogyna	0·204	0·183
2 ,, + L. camara	0·218	0·192
3 ,, + H. eichwaldii	0·196	0·177
4 ,, + C. sparsiflorus	0·182	0·168
5 ,, + C, sat iva	0·166	0·154
6 ,, + X. strumaıium	0·198	0·171
7 ,, + C. oxyacantha	0·224	0·202
8 ,, + C. murale	0·206	0·184
9 ,, + F. Parviflora	0·200	0·176
10 ,, + A. tenuifolius	0·168	0·152
11 ,, + A. arvensis	0·184	0·168
12 ,, + S. arvensis	0·192	0·199
13 ,, + C. album	0·183	0·171
14 ,, + M. indica	0·174	0·159
15 ,, + M. denticulata	0·164	0·155
16 ,, + C. didymus	0·178	0·167
17 ,, + P. lanceolata	0·246	0·214
18 ,, + P. minor	0·168	0·153
19 ,, + C dactylon	0·192	0·175
20 ,, + C. rotundus	0·183	0·157
21 ,, + A. viridus	0·179	0·172

A comparison of table (*a*) and (*b*) would reveal that under sumberged conditions, the amount of exchangeable bases was higher compared to well-drained conditions. It may be related that conductivity also increased more under submerged than under well-drained conditions. It was found that as a result of displacement of K from the exchange complex, the concentration of K^+ in the soil may be almost doubled. Calcium increased from 10 ppm to 138 ppm while Mg from 3·5-5·0 ppm with 0·8% rice straw submerged for 48 days (Clark and Resnicky[4]).

(*d*) Humic acid content during humification of weeds both under well-drained and submerged conditions :

Consequent on the decomposition of weeds added to the soil, the humic acid content increased (table 5). The lignin content of the weeds which is comparatively resistant to the decomposition might have increase the humic acid content. The addition of *P. lanceolata* which has the highest lignin content (15·86%) re ulted in the maximum humic acid content also (0·246% under well-drained and 0·214% under submerged conditions). The data also showed that humic acid content was slightly higher under well-drained as compared to submerged conditions. This may be due to the fact that aeration stimulates organic matter breakdown. Acharya[1] demonstrated that the decomposition of rice straw was most rapid aerobically, slower under water-logged conditions and least pronounced under complete anaerobiasis.

TABLE 6

Effect of weeds on water-stable aggregates, maximum water-holding capacity and sticky point at the end of 120 days period under well-drained conditions

Physical properties / Treatments	Water-stable aggregates %					Maximum water-holding capacity %	Sticky point %
	2mm	2–1mm	1–0.5mm	0.5–0.2mm	Total 0.25mm		
C. Soil alone	0·05	1·24	16·70	13·50	31·49	32·08	9·46
1 Soil + T. monogyna	8·18	6·10	7·72	28·06	50·06	41·14	18·10
2 ,, + L. camara	8·80	7·45	6·44	28·76	51·45	40·82	16·86
3 ,, + H. eichwaldii	6·65	6·90	9·76	27·81	51·12	41·61	17·12
4 ,, + C. sparsiflorus	7·16	9·45	7·92	27·40	51·93	43·38	18·65
5 ,, + C. sativa	9·12	7·16	6·52	33·02	55·82	45·12	19·08
6 ,, + X. strumarium	6·74	9·46	5·83	28·18	50·21	41·19	18·21
7 ,, + C. oxyacantha	5·36	6·90	9·40	28·30	49·96	40·32	16·40
8 ,, + C. murale	11·04	6·80	7·35	31·83	57·02	45·74	19·12
9 ,, + F. parviflora	6·55	7·26	8·48	28·32	50·61	41·88	15·75
10 ,, + A. tenuifolius	8·46	6·95	6·18	33·35	54·94	43·92	17·36
11 ,, + A. arvensis	10·95	7·42	6·21	32·23	56·75	41·10	15·90
12 ,, + S. arvensis	6·72	8·30	11·38	27·46	53·86	42·34	16·88
13 ,, + C. album	12·18	6·10	5·12	35·14	58·54	44·26	20·12
14 ,, + M. indica	7·95	5·86	10·42	27·50	51·73	42·72	17·96
15 ,, + M. denticulata	9·42	8·26	7·75	27·28	52·75	46·38	18·27
16 ,, + C. didymus	6·98	8·18	6·16	28·10	49·36	41·62	15·61
17 ,, + P. lanceolata	6·10	6·72	7·10	30·14	50·06	41·14	16·90
18 ,, + P. minor	9·20	6·13	5·38	32·42	53·13	43·91	16·12
19 ,, + C. dactylon	5·31	7·46	10·34	26·76	49·87	41·78	16·64
20 ,, + C. rotundus	7·46	8·12	6·24	28·98	50·80	40·16	16·88
21 ,, + A. viridus	11·35	5·24	7·40	33·51	57·50	47·64	17·16
Average treatment effect	7·81	6·97	7·99	29·00	51·77	42·28	17·05

(e) Water-stable aggregates, maximum water-holding capacity and sticky point during humification of weeds both under well-drained and submerged conditions.

As a result of the humification of weeds for a period of 120 days, an increase in soil-aggregates greater in size than 0·25 mm was found (table 6). This showed an improvement in soil structure. Comparison of different weeds indicated that C. album formed the highest percentage of soil aggregates greater than 0·25 mm (58·54%) followed by A. viridus, C. album and A. arvensis. This may be due to higher hemicellulose content of these weeds. The organic materials containing more than 11% hemicelluloses and 18% cellulose increased crumb formation. Martin and Anderson[11] and Martin[9,10] have shown that during the decomposition of organic matter large quantities of polysaccharide gums are produced which greatly encourage the formation of water-stable aggregates. It is interesting to record that on analysis, these weeds were found rich in easily decomposable organic constituents like, cellulose and hemicellulose, and accordingly are highly effective in promoting aggregation. On account of the decomposition of weeds, maximum water-holding capacity and sticky point also increased. The humus colloids have the capacity to hold several times its weight of water.

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Influence of Organic matter, Phosphates and Light intensity on Synthesis of amino-acids during carbon nitrogen transformations

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Amino-acids that occur in a free state in soils are of potential importance both to microbial nutrition and plant life. Lochhead and his associates[1,2] have called attention reportedly to a nutritional group of soil bacteria characterized by their dependence on pre-formed amino-acids, while the work of Ghosh and Burris[3] with sterile plants has shown that intact amino-acids can be assimilated by certain higher plants.

For a considerable length of time it was held that amino-acids do not occur in free form in soil (Bremner[4]). The isolation of amino-acids from soil is rendered difficult by the presence of large amounts of extraneous material. However it has now been established that amino-acids exist in the free state in at least some quantities. Dodd, Fowden and Pearsall[5] investigated the presence of free amino-acids in organic soil types using paper-partition chromatography.

The presence of free amino-acids in soil has also been reported by Payne, Rouatt and Katznelson[6], who stated that large numbers of bacteria requiring amino-acids for growth occur in soil. This suggests the possibility that amino-acids exist at least in traces in an uncombined state in soil. The results of these workers suggest that even the mild heat treatment in the concentration of an aqueous soil leachate can prevent the detection of amino-acids which may be present in the free soil solution. Putnam and Schmidt[7], using elution chromatography of concentrated ethanolic extracts, demonstrated the occurrence of free amino-acids in a range of concentration from 2 to 287 μg. per kg.

In view of the above observations an endeavour has been made to identify, separate and estimate the free amino-acids produced during the slow oxidation of different organic materials mixed with sand, TiO_2, ZnO and Fe_2O_3 as surfaces aided by light absorption.

Experimental

100 gms of air dried sand and pure samples (B. D. H. products) of ferric oxide, Zinc oxide and titania were taken in clean white enamelled dishes. To these the energy materials were added to the extent of 0·8% carbon with and without phosphate sources in the form of Tata basic slag and Trichinopoly rock phosphate.

For experimental purposes representative samples were taken out at definite intervals after exposure for the estimation of total nitrogen, identification, separation and estimation of amino-acids.

The procedure for the preparation of the extract for amino-acids was the same as followed by Payne and others[6]. The identication and separation of

amino-acids was undertaken with the help of paper chromatography. The amino acids estimations were carried out according to the colorimetric method of Harding and McLean[8].

The amino-acids which have been identified and separated chromatographically are represented in the following abbreviations :

Glycine = Gly.	Arginine = Ar.	Alanine = Al.
Aspartic acid = Asp.	Valine = Va.	Asparagine = As.
Proline = Pro.	Glutamic acid = Glu.	Histidine = His.
Leucine = Leu.	Lycine = Ly.	Threonine = Threo.

TABLE 1
100 gms of Ganges sand + 0·8% C as Glucose

Period of exposure in days	Total nitrogen mgm.%	Amino-acids identified (chromato-graphically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)	Total Nitrogen mgm.%	Amino-acids identified (chromato-graphically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)
		Light			*Dark*	
0	4·5	–	–	4·5	–	–
30	9·8	Gly, Al, Va, Ly,	0·2745	6·7	Gly, Al, Va,	0·1385
60	12·2	Al, Va, Ly, Glu, Threo.	0·3665	7·9	Gly, Al, Va, Ly	0·1976
90	13·4	Al, Va, Ly, Glu, Threo, Pro.	0·4288	8·6	Al, Va, Ly, Glu, Threo.	0·2595
120	13·9	Al, Va, Ly, Glu, Threo, Pro.	0·4035	8·8	Al, Va, Ly, Glu,	0·2552

TABLE 2
100 gms of Jamuna sand + 0·8% C as Glucose

Period of exposure in days	Total nitrogen mgm.%	Amino-acids identified (chromato-graphically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)	Total Nitrogen mgm.%	Amino-acids identified (chromato-graphically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)
0	4·0	–	–	4·0	–	–
30	10·3	Gly, Al, Va, Glu	0·2987	7·5	Gly, Al, Va	0·1575
60	13·1	Al, Va, Ly, Glu, Threo.	0·4061	7·9	Gly, Al, Va, Ly	0·2054
90	14·5	Al, Va, Ly, Glu, Threo, Pro.	0·4785	8·6	Al, Va, Ly, Glu, Threo.	0·2665
120	15·1	Al, Va, Ly, Glu, Threo, Pro.	0·4681	8·9	Al, Va, Ly, Glu, Threo.	0·2581

TABLE 3
100 gms of Ganges sand + 0·8% C as Glucose + 0·5% P_2O_5 as Tata basic slag

Period of exposure in days	Total nitrogen mgm.%	Amino-acids identified (chromato-graphically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)	Total Nitrogen mgm.%	Amino-acids identified (chromato-graphically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)
0	4·1	–	–	4·1	–	–
30	12·0	Al, Va, Ly, Glu, Threo, Asp	0·6125	7·4	Gly, Al, Va, Ly, Glu	0·3136
60	15·1	Al, Va, Ly, Glu, Threo, Asp, Ar.	0·8312	9·0	Al, Va, Glu, Ly, Threo	0·4235
90	16·5	Va, Ly, Glu, Asp, Ar, Threo, Lue, Pro.	0·9570	9·9	Al, Va, Glu, Ly, Asp, Threo, Leu.	0·4958
120	17·5	Va, Ly, Glu, Asp, Ar, Threo, Leu, Pro.	0·9535	10·4	Al, Va, Glu, Ly, Asp, Threo, Leu.	0·4924

TABLE 4

100 gms of Jamuna sand + 0·8% C as Glucose + 0·5% P_2O_5 as Tata basic slag

Period of exposure in days	Total nitrogen mgm.%	Amino-acids identified (chromatographically)	Amount of amino-acid with respect to glycine mgm.% (colorimetrically)	Total nitrogen mgm.%	Amino-acids identified (chromatographically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)
		Light			*Dark*	
0	3·7	—	—	3·7	—	—
30	12·8	Al, Va, Ly, Glu, Threo, Asp.	0·6653	7·6	Gly, Al, Va, Ly, Glu.	0·3345
60	16·2	Al, Va, Ly, Glu, Threo, Asp, Ar.	0·9235	9·5	Al, Va, Glu, Ly, Threo.	0·4659
90	17·9	Va, Ly, Glu, Asp, Threo, Ar, Leu, Pro.	1·0735	10·4	Al, Va, Glu, Ly, Asp, Threo, Leu.	0·5415
120	19·0	Va, Ly, Glu, Asp, Ar, Threo, Leu, Pro.	1·0702	11·0	Al, Va, Glu, Ly, Asp, Threo, Leu.	0·5398

TABLE 5

100 gms of Ganges sand + 0·8% C as Glucose + 0·5% P_2O_5 as Trichinopoly rock phosphate

Period of exposure in days	Total nitrogen mgm.%	Amino-acids identified (chromatographically)	Amount of amino-acid with respect to glycine mgm.% (colorimetrically)	Total nitrogen mgm.%	Amino-acids identified (chromatographically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)
0	4·4	—	—	4·4	—	—
30	11·1	Al, Va, Glu, Ly, Asp, Threo.	0·4995	7·2	Ly, Gly, Al, Va, Glu.	0·2738
60	14·0	Al, Va, Glu, Ly, Asp, Threo.	0·7112	8·6	Ly, Al, Va, Glu, Threo.	0·3695
90	15·4	Al, Va, Glu, Ly, Asp, Threo, Leu.	0·8164	9·3	Ly, Al, Va, Glu, Threo.	0·4279
120	16·1	Al, Va, Glu, Ly, Asp, Threo, Leu.	0·7995	9·8	Ly, Al, Va, Glu, Threo.	0·4125

TABLE 6

100 gms of Jamuna sand + 0·8% C as Glucose + 0·5% P_2O_5 as Trichinopoly rock phosphate

Period of exposure in days	Total nitrogen mgm.%	Amino-acids identified (chromatographically)	Amount of amino-acid with respect to glycine mgm.% (colorimetrically)	Total nitrogen mgm.%	Amino-acids identified (chromatographically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)
0	3·9	—	—	3·9	—	—
30	11·8	Al, Va, Glu, Ly, Asp, Threo.	0·5549	7·1	Ly, Gly, Al, Va, Glu.	0·2845
60	14·8	Al, Va, Glu, Ly, Asp, Threo.	0·7695	8·7	Ly, Al, Va, Glu, Threo, Asp.	0·3923
90	16·3	Al, Va, Glu, Ly, Asp, Threo, Leu.	0·8968	9·5	Ly, Al, Va, Glu, Threo, Asp.	0·4565
120	17·3	Al, Va, Glu, Ly, As, Threo, Leu	0·8823	10·0	Ly, Al, Va, Glu, Threo. Asp.	0·4426

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TABLE 7
100 gms of Ganges sand + 0·8% C as Wheat straw

Period of exposure in days	Total nitrogen mgm.%	Amino-acids identified (chromatographically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)	Total nitrogen mgm.%	Amino-acids identified (chromatographically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)
		Light			*Dark*	
0	17·1	–	–	17·1	–	–
60	28·2	Al,Va,Glu,Asp,Ly.	0·4645	19·7	Al, Glu, Va, Asp,	0·3542
120	25·5	Al, Va, Glu, Asp, Ly, Threo, Leu.	0·8928	20·9	Al, Glu, Va, Asp, Ly, Threo.	0·6275
180	26·4	Al, Va, Glu, Asp, Ly, Threo, Leu.	ı·1085	21·4	Al, Glu, Va, Asp, Ly, Threo.	0·7706

TABLE 8
100 gms of Jamuna sand + 0·8% C as Wheat straw

Period of exposure in days	Total nitrogen mgm.%	Amino-acids identified (chromatographically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)	Total nitrogen mgm.%	Amino-acids identified (chromatographically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)
0	16·6	–	–	16·6	–	–
60	23·6	Al,Va,Glu,Asp,Ly	0·5196	19·5	Al, Glu, Va, Asp.	0·3905
120	26·1	Al, Va, Glu, Asp, Ly, Threo, Leu.	0·9658	21·1	Al, Glu, Va, Asp, Ly, Threo.	0·6728
180	180	Al, Va, Glu, Asp, Ly, Threo, Leu.	1·2065	21·5	Al, Glu, Va, Asp, Ly, Threo.	0·0416

TABLE 9
100 gms of Ganges sand + 0·8% C as Wheat straw + 0·5% as P_2O_5 as Tata basic slag

Period of exposure in days	Total nitrogen mgm.%	Amino-acids identified (chromatographically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)	Total nitrogen mgm.%	Amino-acids identified (chromatographically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)
0	16·1	–	–	16·1	–	–
60	25·0	Al, Va, Ly, Asp, Glu,Asp,Threo,Pro.	1·0556	20·0	Al, Va, Ly, Glu, Asp, Threo.	0·7015
120	28·0	Ly, Va, As, Glu, As, Ar, His, Threo, Leu.	1·5965	21·5	Al, Va, Ly, Glu, Asp,Ar,His,Threo.	1·0796
180	29·4	Ly, Va, Asp, Glu, As, Ar, His, Threo, Leu.	2·1058	22·2	Al, Va, Ly, Glu, Asp, Ar, His, Threo.	1·4215

TABLE 10

100 gms of Jamuna sand + 0·8% C as Wheat straw + 0·5% P_2O_5 as Tata basic slag

Period of exposure in days	Total nitrogen mgm.%	Amino-acids identified (chromato-graphically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)	Total nitrogen mgm.%	Amino acids identified (chromato-graphically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)
		Light			*Dark*	
0	15·6	–	–	15·6	–	–
60	25·8	Al, Va, Ly, Asp, Glu, As, Threo,Pro.	1·1352	20·2	Al, Va, Ly, Glu, Asp, Threo.	0·7475
120	29·1	Ly, Va, Asp, Glu, As, Ar, His, Threo, Leu.	1·7169	21·9	Al, Va, Ly, Glu, Asp, Ar, His, Threo.	1·1393
180	30·5	Ly, Va, Asp, Glu, As, Ar, His, Threo, Leu.	2·1965	22·7	Al, Va, Ly, Glu, Asp, Ar, His, Threo.	1·4755

TABLE 11

100 gms of Ganges sand + 0·8% C as Wheat straw + 0·5% P_2O_5 as Trichinopoly rock phosphate

Period of exposure in days	Total nitrogen mgm.%	Amino-acids identified (chromato-graphically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)	Total nitrogen mgm.%	Amino acids identified (chromato-graphically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)
0	16·8	–	–	16·8	–	–
60	24·6	Al, Va, Ly, Glu, Asp, Threo.	0·9105	20·3	Al, Va, Ly, Glu, Asp.	0·6295
120	27·3	Al, Va, Ly, Glu, Asp, Threo, Pro, Leu.	1·3925	21·7	Al, Va, Ly, Glu, Asp, Threo.	0·9768
180	28·6	Al, Va, Ly, Glu, Asp, Threo, Pro, Leu.	1·8014	22·3	Al, Va, Ly, Glu, Asp, Threo, Leu.	1·2488

TABLE 12

100 gms of Jamuna sand + 0·8% C as Wheat straw + 0·5% P_2O_5 as Trichinopoly rock phosphate

Period of exposure in days	Total nitrogen mgm.%	Amino-acids identified (chromato-graphically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)	Total nitrogen mgm.%	Amino acids identified (chromato-graphically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)
0	16·3	–	–	16·3	–	–
60	25·3	Al, Va, Ly, Gu, Asp, Threo.	0·9865	20·2	Al, Va, Ly, Glu, Asp.	0·6468
120	28·3	Al, Va, Ly, Glu, Asp,Threo,His,Ar.	1·4996	21·8	Al, Va, Ly, Glu, Asp, Threo, His.	1·0245
180	29·7	Al, Va, Ly, Glu, Asp,Threo,His,Ar.	1·9615	22·5	Al, Va, Ly, Glu, Asp. Threo, His.	1·3056

TABLE 13

100 gms of Ganges sand + 0·8% C as Saw dust

Period of exposure in days	Total nitrogen mgm.%	Amino-acids identified (chromato-graphically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)	Total nitrogen mgm.%	Amino acids identified (chromato-graphically)	Total amino-acids with respect to glycine mgm.% (colorimetrically)
		Light			*Dark*	
0	12·6	–	–	12·6	–	–
60	16·8	Gly, Al, Va, Glu.	0·3025	14·3	Gly, Al, Va.	0·2445
120	18·3	Gly, Al, Va, Glu, Asp, Threo.	0·6438	15·2	Glu, Al, Va, Gly, Threo.	0·4418
180	19·0	Al, Va, Glu, Asp, Threo, Leu.	0·7615	15·6	Gly, Al, Va, Glu, Threo, Leu.	0·5465

TABLE 14

100 gms of Jamuna sand + 0·8% C as Saw dust

0	12·1	–	–	12·1	–	–
60	16·6	Gly, Al, Va, Glu,	0·3488	14·0	Gly, Al, Va.	0·2665
120	18·2	Gly, Al, Va, Glu, Asp, Threo.	0·6556	15·0	Glu, Al, Va, Gly, Threo.	0·4653
180	19·4	Al, Va, Glu, Asp, Threo, Leu.	0·8345	15·5	Gly, Al, Va, Glu, Threo.	0·5735

TABLE 15

100 gms of Ganges sand + 0·8% C as Sawdust + 0·5% P_2O_5 as Tata basic slag

0	11·9	–	–	11·9	–	–
60	18·2	Al, Va, Glu, Asp, Threo, Leu.	0·7285	14·6	Al, Va, Glu, Asp, Threo.	0·4968
120	20·6	Al, Va, Glu, Asp, Threo, Leu.	1·1338	15·8	Al, Va, Glu, Asp, Threo, Leu.	0·7745
180	21·8	Al, Va, Glu, Asp, Ar, As, Threo, Leu.	1·4825	16·4	Al, Va, Glu, Asp, Ar, Threo, Leu.	1·0236

TABLE 16

100 gms of Jamuna sand + 0·8% C as Sawdust + 0·5% P_2O_5 as Tata basic slag

0	11·4	–	–	11·4	–	–
60	18·5	Al, Va, Glu, Asp, Threo, Leu.	0·7915	14·5	Al, Va, Glu, Asp, Threo.	0·5225
120	21·1	Al, Va, Glu, Asp, Threo, Leu.	1·2236	15·8	Al, Va, Glu, Asp, Threo, Leu,	0·8089
180	22·5	Al, Va, Glu, Asp, Threo, Leu, Ar, As.	1·5975	16·5	Al, Va, Glu, Asp, Threo, Ar, Leu.	1·0565

TABLE 17

100 gms of Ganges sand + 0·8% C as Sawdust + 0·5% P$_2$O$_5$ as Trichinopoly rock phosphate

Period of exposure in days	Total nitrogen mgm.%	Amino acids identified (chromato-graphically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)	Total nitrogen mgm.%	Amino-acids identified (chromato-graphically)	Amount of amino-acids with respect to glycine mgm % (colorimetrically)
		Light			*Dark*	
0	12·4	–	–	12·4	–	–
60	17·6	Al, Va, Asp, Glu, Threo.	0·6165	14·6	Al, Va, Asp, Glu.	0·4384
120	19·7	Al, Va, Asp, Glu, Threo, Leu, Ar.	0·9654	15·7	Al, Va, Glu, Asp, Threo, Leu.	0·6908
180	20·6	Al, Va, Asp, Glu, Threo, Leu, Ar.	1·2566	16·2	Al, Va, Glu, Asp, Threo, Leu.	0·8915

TABLE 18

100 gms of Jamuna sand + 0·8% C as Sawdust + 0·5% P$_2$O$_5$ as Trichinopoly rock phosphate

Period of exposure in days	Total nitrogen mgm.%	Amino acids identified (chromato-graphically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)	Total nitrogen mgm.%	Amino-acids identified (chromato-graphically)	Amount of amino-acids with respect to glycine mgm % (colorimetrically)
0	11·9	–	–	11·9	–	–
60	17·8	Al, Va, Glu, Asp, Threo.	0·6769	14·4	Al, Va, Glu, Asp.	0.4465
120	20·1	Al, Va, Glu, Asp, Threo, Leu, Ar.	1·0452	15·5	Al, Va, Glu, Asp, Threo, Leu.	0·7136
180	21·3	Al, Va, Glu, Asp, Threo, Leu, Ar.	1·03845	16·1	Al, Va, Glu, Asp, Threo, Leu.	0·9018

TABLE 19

100 gms of Zinc oxide + 0·8% C as Glucose

Period of exposure in days	Total nitrogen mgm.%	Amino acids identified (chromato-graphically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)	Total nitrogen mgm.%	Amino-acids identified (chromato-graphically)	Amount of amino-acids with respect to glycine mgm % (colorimetrically)
0	–	–	–	–	–	–
30	7·2	Gly, Al, Va, Ly.	0·2088	2·9	Gly, Al, Va.	0·0579
60	10·1	Gly, Al, Va, Ly, Threo, Pro.	0·3135	4·4	Al, Va, Gly, Ly, Threo.	0·1145
90	11·7	Al, Va, Ly, Threo, Pro, Leu.	0·3862	5·2	Al, Va, Ly, Threo, Pro.	0·1612
120	12·4	Al, Va, Ly, Threo, Pro, Leu.	0·3726	5·5	Al, Va, Ly, Threo, Pro.	0·1548

TABLE 20

100 gms of Zinc oxide + 0·8% C as Glucose + 0·5% P$_2$O$_5$ as Tata basic slag

Period of exposure in days	Total nitrogen mgm.%	Amino acids identified (chromato-graphically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)	Total nitrogen mgm.%	Amino-acids identified (chromato-graphically)	Amount of amino-acids with respect to glycine mgm % (colorimetrically)
0	–	–	–	–	–	–
30	11·0	Al, Va, Gly, Threo, Ly.	0·5725	4·8	Al, Va, Gly, Threo.	0·2125
60	14·8	Al, Va, Threo, Ly, Pro, Glu, Leu.	0·8436	6·8	Al, Va, Threo, Ly, Pro, Glu.	0·3354
90	16·7	Al, Va, Threo, Ly, Glu, Leu, Asp.	1·0156	7·8	Al, Va, Threo, Ly, Pro, Glu, Leu.	0·4056
120	17·9	Al, Va, Threo, Ly, Pro, Glu, Leu, Asp.	1·0105	8·5	Al, Va, Threo, Ly, Pro, Glu, Leu.	0·4015

TABLE 21
100 gms of Zinc oxide + 0·8% C as Glucose + 0·5% P_2O_5 as Trichinopoly rock phosphate

Period of exposure in days	Total nitrogen mgm.%	Amino-acids identified (chromatographically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)	Total nitrogen mgm.%	Amino-acids identified (chromatographically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)
		Light			*Dark*	
0	—	—	—	—	—	—
0	9·4	Al,Va,Threo,Gly.	0·4415	4·0	Al,Va,Glu,Threo.	0·1636
60	12·7	Al, Va, Threo, Ly, Pro, Leu, Glu.	0·6606	5·7	Al, Va, Threo, Ly, Pro. Leu.	0·2565
90	14·4	Al, Va, Threo, Ly Pro, Leu Glu, Asp.	0·7920	6·7	Al, Va, Threo, Ly, Pro. Leu, Glu.	0·3218
120	15·6	Al, Va, Threo, Ly Pro.Leu,Glu, Asp.	0·7804	7·3	Al, Va, Threo, Ly, Pro, Leu, Glu.	0·3161

TABLE 22
100 gms of Ferric oxide + 0·8% C as Glucose

Period of exposure in days	Total nitrogen mgm.%	Amino-acids identified (chromatographically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)	Total nitrogen mgm.%	Amino-acids identified (chromatographically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)
0	—	—	—	—	—	—
30	6·4	Al, Gly, Va, Ly.	0·1795	2·6	Al, Gly, Va,	0·0525
60	9·2	Al, Gly, Va, Ly, Threo.	0·2759	4·1	Al, Gly, Va, Ly, Threo.	0·1018
90	10·7	Al, Va, Ly, Threo, Pro.	0·3462	4·9	Al, Gly, Va, Ly, Threo.	0·1475
120	11·3	Al, Va, Ly, Threo, Pro.	0·3277	5·2	Al, Va, Ly, Threo, Gly.	0·1404

TABLE 23
100 gms of Ferric oxide + 0·8% C as Glucose + 0·5% P_2O_5 as Tata basic slag

Period of exposure in days	Total nitrogen mgm.%	Amino-acids identified (chromatographically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)	Total nitrogen mgm.%	Amino-acids identified (chromatographically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)
0	—	—	—	—	—	—
30	10·1	Ly, Al, Va, Gly, Threo.	0·5145	4·4	Al, Va, Gly, Threo.	0·1849
60	13·8	Ly,Al,Va,Threo, Pro, Glu, Leu.	0·7596	6·3	Al, Va, Threo, Ly, Pro, Glu.	0·2965
90	15·7	Threo, Pro, Glu, Leu, Ly,Al, Va.	0·9106	7·3	Al, Va, Threo, Ly, Pro, Glu.	0·3655
120	16·8	Ly,Al,Va,Threo, Pro. Glu, Leu.	0·9045	7·9	Al, Va, Threo, Ly, Pro, Glu.	0·3613

TABLE 24

100 gms of Ferric oxide + 0.8% C as Glucose + 0.5% P_2O_5 as Trichinopoly rock phosphate

Period of exposure in days	Total nitrogen mgm.%	Amino-acids identified (chromatographically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)	Total nitrogen mgm.%	Amino-acids identified (chromatographically)	Amount of amino-acids with respect to Glycine mgm.% (colorimetrically)
		Light			*Dark*	
0 30	8.6	Al, Va, Gly, Threo, Ly.	0.3872	3.6	Al, Va, Gly, Threo.	0.1368
60	11.8	Al, Va, Threo, Pro, Ly, Leu.	0.5932	5.2	Al, Va, Threo, Ly, Pro.	0.2236
90	13.5	Al, Va, Threo, Pro, Ly, Leu, Glu.	0.7155	6.2	Al, Va, Threo, Ly, Pro, Glu.	0.2852
120	14.5	Al, Va, Threo, Pro, Leu, Glu.	0.7031	6.8	Al, Va, Threo, Ly, Pro, Glu.	0.2791

TABLE 25

100 gms of Titania + 0.8% C as Glucose

0 30	7.5	Al, Gly, Va, Ly,	0.2246	3.1	Al, Gly, Va.	0.0685
60	10.5	Al, Gly, Va, Ly, Threo, Pro.	0.3367	4.7	Al, Gly, Va, Ly, Threo.	0.1269
90	12.1	Al, Va, Ly, Threo, Pro, Leu.	0.4115	5.5	Al, Va, Ly, Threo, Pro.	0.1764
120	12.8	Al, Va, Ly, Threo, Pro, Leu.	0.3985	5.9	Al, Va, Ly, Threo, Pro.	0.1702

TABLE 26

100 gms of Titania + 0.8% C as Glucose + 0.5% P_2O_5 as Tata basic slag

0 30	11.4	Al, Va, Gly, Threo, Ly.	0.6045	5.0	Al, Va, Threo, Gly.	0.2653
60	15.2	Al, Va, Threo, Ly, Pro, Glu, Leu.	0.8818	7.0	Al, Va, Threo, Ly, Pro. Glu, Leu.	0.4064
90	17.2	Al, Va, Threo, Ly, Pro, Glu, Leu, Asp.	1.0492	8.1	Al, Va, Threo, Ly, Pro, Glu, Leu.	0.5105
120	18.4	Al, Va, Threo, Ly, Pro, Glu, Leu, Asp.	1.0445	8.8	Al, Va, Threo, Ly, Pro, Glu, Leu	0.5066

TABLE 27

100 gms of Titania + 0·8% C as Glucose + 0·5% P_2O_5 as Trichinopoly rock phosphate

Period of exposure in days	Total nitrogen mgm.%	Amino-acids identified (chromato-graphically)	Amount of amino acids with respect to glycine mgm.% (colorimet-rically)	Total nitrogen mgm.%	Amino-acids identified (chromato-graphically)	Amount of amino-acids with respect to glycine mgm.% (colorimet-rically)
		Light			*Dark*	
0	—	—	—	—	—	—
30	9·8	Al, Va, Gly, Threo, Ly.	0·4802	4·2	Al, Va, Gly, Threo.	0·1785
60	13·2	Al, Va, Threo, Ly, Pro, Leu, Glu.	0·7128	5·9	Al, Va, Threo, Ly, Pro, Leu.	0·2832
90	15·0	Al, Va, Threo, Ly, Pro, Leu, Glu, Asp.	0·8545	7·0	Al, Va, Threo, Ly, Pro, Leu, Glu.	0·3576
120	16·2	Al, Va, Threo, Ly, Pro, Leu, Glu, Asp.	0·8424	7·6	Al, Va, Threo, Ly, Pro, Leu, Glu.	0·3522

TABLE 28

100 gms of Zinc oxide + 0·8% C as Wheat straw

Period of exposure in days	Total nitrogen mgm.%	Amino-acids identified (chromato-graphically)	Amount of amino acids with respect to glycine mgm.% (colorimet-rically)	Total nitrogen mgm.%	Amino-acids identified (chromato-graphically)	Amount of amino-acids with respect to glycine mgm.% (colorimet-rically)
0	12·6	—	—	12·6	—	—
60	20·6	Ly, Va, Glu, Al, Gly, Asp.	0·4535	16·0	Ly, Glu, Va, Al, Gly.	0·3196
120	23·3	Ly, Glu, Va, Al, Threo, Asp, Leu.	0·8624	17·8	Ly, Glu, Va, Al, Threo, Asp.	0·5698
180	24·7	Ly, Glu, Va, Al, Threo, Asp, Leu.	1·0869	18·3	Ly, Glu, Va, Al, Threo, Asp.	0·6054

TABLE 29

100 gms of Zinc oxide + 0·8% C as Wheat straw + 0·5% P_2O_5 as Tata basic slag

Period of exposure in days	Total nitrogen mgm.%	Amino-acids identified (chromato-graphically)	Amount of amino acids with respect to glycine mgm.% (colorimet-rically)	Total nitrogen mgm.%	Amino-acids identified (chromato-graphically)	Amount of amino-acids with respect to glycine mgm.% (colorimet-rically)
0	11·8	—	—	11·8	—	—
60	23·7	Ly, Asp, Glu, Va, Al, Threo, Pro.	1·0428	17·0	Ly, Asp, Glu, Va, Al, Threo.	0·6295
120	27·2	Ly, Asp, Glu, Va, Al, Threo, Pro, As, His.	1·6046	18·9	Ly, Asp, Glu, Va, Al, Threo, Pro.	0·9828
180	28·7	Ly, Asp, Glu, Va, Al, Threo, Pro, As, His.	2·0665	19·7	Ly, Asp, Glu, Va, Al, Threo, Pro, His.	1·2806

TABLE 30

100 gms of Zinc oxide + 0·8% C as Wheat straw + 0·5% P_2O_5 as Trichinopoly rock phosphate

Period of exposure in days	Total nitrogen mgm.%	Amino-acids identified (chromatographically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)	Total nitrogen mgm.%	Amino-acids identified (chromatographically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)
		Light			Dark	
0	12·4	–	–	12·4	–	–
60	22·8	Ly, Asp, Glu, Va, Al, Threo.	0·8895	16·9	Ly, Asp, Glu, Va, Al.	0·5409
120	26·0	Ly, Asp, Glu,Va, Al,Threo, Pro.	1·3790	18·7	Ly,Asp,Glu,Va, Al, Threo, Pro.	0·8788
180	27·5	Ly, Asp, Glu, Va, Al,Threo, Pro,Leu.	1·8150	19·5	Ly, Asp,Glu,Va, Al,ThreoPro.	1·1325

TABLE 31

100 gms of Ferric oxide + 0·8% C as Wheat straw

0	12·6	–	–	12·6	–	–
60	19·8	Gly, Al,Va, Glu, Ly.	0·3973	15·6	Al, Gly, Ly, Va.	0·2809
120	22·4	Al, Va, Glu, Ly, Threo, Asp.	0·7835	17·3	Gly, Al, Va, Ly, Threo, Glu.	0·5186
180	23·7	Al, Va, Glu, Ly, Threo, Asp.	0·9958	17·8	Gly, Al, Va, Ly, Threo, Glu.	0·6408

TABLE 32

100 gms of Ferric oxide + 0·8% C as Wheat straw + 0·5% P_2O_5 as Tata basic slag

0	11·8	–	–	11·8	–	–
60	22·7	Ly, Asp, Va, Glu,Al,Threo.Pro.	0·9538	16·6	Ly, Asp, Glu, Va, Al, Threo.	0·5815
120	26·1	Ly,Asp, Glu, Va, Al, Threo, Pro, As.	1·4876	18·5	Ly, Asp, Glu,Va, Al, Threo, Pro.	0·9256
180	27·5	Ly, Asp, Glu, Va, Al, Threo, Pro, His, As.	1·9251	19·3	Ly, Asp,Glu, Va, Al, Threo, Pro, His.	1·2348

TABLE 33

100 gms of Ferric oxide + 0·8% C as Wheat straw + 0·5% P_2O_5 as Trichinopoly rock phosphate

0	12·4	–	–	12·4	–	–
60	21·9	Ly, Asp, Glu, Va, Al, Threo.	0·8103	16·5	Ly, Asp, Glu, Al, Va.	0·5116
120	25·0	Ly, Asp, Glu, Va, Al, Threo, Pro.	1·2752	18·2	Ly, Asp, Glu, Al, Va, Threo.	0·8796
180	26·4	Ly, Asp, Glu, Va,Al, Threo, Pro,Leu.	1·6596	18·9	Ly, Asp, Glu,Va, Al, Threo, Pro.	1·0534

TABLE 34

100 gms of Titania + 0·8% C as Wheat straw

Period of exposure in days	Total nitrogen mgm.%	Amino-acid identified (chromato-graphically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)	Total nitrogen mgm.%	Amino-acids identified (chromato-graphically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)
		Light			*Dark*	
0	12·6	—	—	12·6	—	—
60	20·9	Ly, Glu, Gly, Al, Asp.	0·4807	16·1	Ly, Glu, Va, Al, Gly.	0·3542
120	23·7	Ly, Gly, Va, Al, Threo, Asp, Leu.	1·8998	18·0	Ly, Glu, Va, Al, Asp, Threo.	0·6125
180	25·1	Ly, Glu, Va Al, Threo, Asp, Leu.	1·1549	18·5	Ly, Glu, Va, Al, Asp, Threo.	0·7406

TABLE 35

100 gms of Titania + 0·8% C as Wheat straw + 0·5% P₂O₅ as Tata basic slag

Period of exposure in days	Total nitrogen mgm.%	Amino-acid identified	Amount of amino-acids with respect to glycine	Total nitrogen mgm.%	Amino-acids identified	Amount of amino-acids with respect to glycine
0	11·8	—	—	11·8	—	—
60	24·1	Ly, Asp, Glu, Va, Ar, As, Threo, Pro.	1·1246	17·2	Ly, Asp, Glu, Va, Al. Threo, Pro.	0·6875
120	27·7	Ly, Asp, Glu, Va, As, Ar, Threo, Pro, His.	1·6683	19·2	Ly, Asp, Glu, Va, Al, Threo, Pro.	1·0367
180	29·2	Ly, Asp, Glu, Va, As, Ar, Threo, Pro, His.	2·1602	20·1	Ly, Asp, Glu, Va, Al, Threo, Pro, His.	1·3459

TABLE 36

100 gms of Titania + 0·8% C as Wheat straw + 0·5% P₂O₅ as Trichinopoly rock phosphate

Period of exposure in days	Total nitrogen mgm.%	Amino-acid identified	Amount of amino-acids with respect to glycine	Total nitrogen mgm.%	Amino-acids identified	Amount of amino-acids with respect to glycine
0	12·4	—	—	12·4	—	—
60	23·3	Al, Va, Ly, Glu, Threo, Asp.	0·9554	17·1	Al, Ly, Va, Glu, Asp, Threo.	0·5815
120	26·6	Al, Ly, Va, Asp, Glu, Threo, His, Pro.	1·4631	18·9	Al, Ly, Va, Asp, Glu, Threo, Pro.	0·9456
180	28·2	Al, Ly, Va, Asp, Glu, Threo, Pro.	1·9176	19·8	Al, Ly, Va, Asp, Glu, Threo, Pro.	1·1882

TABLE 37

100 gms of Zinc oxide + 0·8% C as Sawdust

Period of exposure in days	Total nitrogen mgm.%	Amino-acid identified	Amount of amino-acids with respect to glycine	Total nitrogen mgm.%	Amino-acids identified	Amount of amino-acids with respect to glycine
0	8·4	—	—	8·4	—	—
60	13·7	Al, Gly, Va, Threo.	0·2877	10·6	Al, Gly, Va	0·2125
120	15·5	Al, Glu, Va, Threo, Leu.	0·5578	11·8	Al, Gly, Va, Threo, Leu.	0·3658
180	16·9	Al, Glu, Va, Threo, Leu.	0·7269	12·3	Al, Gly, Va, Threo, Leu.	0·4552

TABLE 38

100 gms of Zinc oxide + 0·8% C as Sawdust + 0·5% P_2O_5 as Tata basic slag

Period of exposure in days	Total nitrogen mgm.%	Amino-acids identified (chromatographically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)	Total nitrogen mgm.%	Amino-acids identified (chromatographically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)
		Light			*Dark*	
0	7·9	–	–	7·9	–	–
60	16·4	Al, Va, Glu, Asp, Threo.	0·7056	11·5	Al, Va, Glu, Asp.	0·4145
120	19·2	Al. Va, Asp, Glu, Threo, Leu.	1·1133	13·0	Al, Va, Glu, Asp, Threo.	0·6638
180	20·8	Al, Va, Asp, Glu, Threo, Leu.	1 4769	13·7	Al, Va, Glu, Asp, Threo.	0·8769

TABLE 39

100 gms of Zinc oxide + 0·8% C as Sawdust + 0·5% P_2O_5 as Trichinopoly rock phosphate

Period of exposure in days	Total nitrogen mgm.%	Amino-acids identified (chromatographically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)	Total nitrogen mgm.%	Amino-acids identified (chromatographically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)
0	8·3	–	–	8·3	–	–
60	15·4	Gly, Al, Glu, Va, Threo.	0·5859	11·3	Al, Gly, Va, Glu.	0·3503
120	17·9	Al, Glu, Va, Threo, Asp.	0·9808	12·5	Al, Gly, Glu, Va, Threo.	0·5753
180	19·2	Al, Glu, Va, Threo, Asp, Leu.	1·2485	13·2	Al, Glu, Va, Threo, Asp.	0·7395

TABLE 40

100 gms of Ferric oxide + 0·8% C as Sawdust

Period of exposure in days	Total nitrogen mgm.%	Amino-acids identified (chromatographically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)	Total nitrogen mgm.%	Amino-acids identified (chromatographically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)
0	8·4	–	–	8·4	–	–
60	13·1	Al, Va, Gly, Threo.	0·2358	10·4	Al, Va, Gly.	0·1768
120	14·8	Al, Va, Gly, Threo.	0·4985	11·5	Al, Va, Gly, Threo.	0·3335
180	16·0	Al, Va, Gly, Threo, Leu.	0·6413	11·9	Al, Va, Gly, Threo.	0·4168

TABLE 41

100 gms of Ferric oxide + 0·8% C as Sawdust + 0·5% P_2O_5 as Tata basic slag

Period of exposure in days	Total nitrogen mgm.%	Amino-acids identified (chromatographically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)	Total nitrogen mgm.%	Amino-acids identified (chromatographically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)
0	7·9	–	–	7·9	–	–
60	15·7	Al, Va, Glu, Threo.	0·6286	11·0	Al, Va, Glu, Asp.	0·3815
120	18·4	Al, Va, Asp, Glu, Leu, Threo.	1·0125	12·6	Al, Va, Glu, Asp, Threo.	0·6178
180	19·9	Al, Va, Asp, Glu, Leu, Threo.	1·3538	13·3	Al, Va, Glu, Asp, Threo.	0·8379

TABLE 42

100 gms of Ferric oxide + 0·8% C as Sawdust + 0·5% P_2O_5 as Trichinopoly rock phosphate

Period of exposure in days	Total nitrogen mgm.%	Amino-acids identified (chromato-graphically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)	Total nitrogen mgm.%	Amino-acids identified (chromato-graphically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)
		Light			*Dark*	
0	8·3	—	—	8·3	—	—
60	14·7	Al, Gly, Glu, Va, Threo.	0·5146	11·0	Al, Gly, Va, Glu,	0·3815
120	17·1	Al, Glu, Va, Threo, Asp.	0·8379	12·2	Al, Gly, Glu, Va, Threo.	0·5368
180	18·3	Al, Glu, Va, Threo, Asp, Leu,	1·1041	12·8	Al, Glu, Va, Asp, Threo.	0·7035

TABLE 43

100 gms of Titania + 0·8% C as Sawdust

Period of exposure in days	Total nitrogen mgm.%	Amino-acids identified (chromato-graphically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)	Total nitrogen mgm.%	Amino-acids identified (chromato-graphically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)
0	8·4	—	—	8·4	—	—
60	14·0	Al, Va, Threo, Glu.	0·3125	10·8	Al, Va, Gly, Threo.	0·2168
120	15·8	Al, Va, Threo, Glu, Leu, Asp.	0·5846	12·0	Al, Va, Threo, Glu, Leu.	0·3858
180	17·1	Al, Va, Threo, Glu, Leu, Asp,	0·7528	12·5	Al, Va, Threo. Glu, Leu.	0·5105

TABLE 44

100 gms of Titania + 0·8% C as Sawdust + 0·5% P_2O_5 as Tata basic slag

Period of exposure in days	Total nitrogen mgm.%	Amino-acids identified (chromato-graphically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)	Total nitrogen mgm.%	Amino-acids identified (chromato-graphically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)
0	7·9	—	—	7·9	—	—
60	16·8	Al, Va, Glu, Asp, Threo, Leu.	0·6392	11·7	Al, Va, Glu, Asp, Threo.	0·4329
120	19·7	Al, Va, Glu, Asp, Threo, Leu, Ar.	1·1625	13·2	Al, Va, Glu, Asp, Threo, Leu.	0·6864
180	21·3	Al, Va, Glu, Asp, Threo, Leu, Ar.	1·5548	13·9	Al, Va, Glu, Asp, Threo, Leu.	0·9035

TABLE 45

100 gms of Titania + 0·8% C as Sawdust + 0·5% P_2O_5 as Trichinopoly rock phosphate

Period of exposure in days	Total nitrogen mgm.%	Amino-acids identified (chromato-graphically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)	Total nitrogen mgm.%	Amino-acids identified (chromato-graphically)	Amount of amino-acids with respect to glycine mgm.% (colorimetrically)
0	8·3	—	—	8·3	—	—
60	15·7	Al, Glu, Va, Asp, Threo.	0·6285	11·5	Al, Va, Glu, Threo.	0·3568
120	18·3	Al, Glu, Va, Asp, Threo, Leu.	0·9882	12·8	Al, Va, Glu, Threo, Leu.	0·6144
180	19·7	Al, Glu, Va, Asp, Threo, Leu, Ar.	1·3199	13·5	Al, Va, Threo, Glu, Leu, Asp.	0·7838

Discussion

The experimental results recorded in the foregoing pages show that when organic materials, like Glucose, Wheat straw and Sawdust are mixed with sand or metallic oxides as surface, amino-acids are produced in small amounts. It is also observed that the number as well as the amount of amino-acids produced is greater in the sets exposed to light than in those kept in the dark.

It is observed that the rate of increase in total nitrogen is in the following order with different energy materials utilised :

$$Glucose > Wheat\ straw > Sawdust$$

From these observations it is clear that the increase in nitrogen is directly related to the oxidation of carbon, since the oxidation of carbon also takes place in the same order with the said energy materials[9]. With respect to metallic oxides as sarfaces, it is of the following order :

$$TiO_2 > ZnO > Fe_2O_3.$$

The difference in activity with different metallic oxides as surfaces is due to their difference in the power of Photo-sensitization, showing thereby the marked influence of light absorption on nitrogen fixation and the subsequent utilisation of fixed nitrogen in the formation of amino-acids.

From our experimental results it is also observed that in the systems containing Glucose as energy material, the amount of amino-acids gradually increases in the beginning but shows a slight decrease at the final stage. On the other hand, in the systems with Wheat straw or Sawdust as energy materials, the amino-acids appear gradually and instead of decrease at the final stage, the amount of amino-acids is greatly increased. The observations show that in the systems with Glucose, the amino-acids are synthesised from the fixed nitrogen formed by the oxidation of the energy material aided by the absorption of light, since there is no original protein or amino-acid in the material and the decrease of amino-acids can be attributed to the fact that amino-acids being readily oxidisable undergo ammonification and nitrification aided by light radiations and the net result being a tendency towards loss at the final stage. On the other hand, in the systems with Wheat straw or Sawdust, the amino-acids may be partly derived from the protein originally present in these energy material, because, the original protein of the materials can undergo hydrolysis in the course of time with the result that amino-acids derived from the proteins show their presence in large quantities at the final stage.

It is very interesting to note from our experimental results that some amino-acids which are formed in the beginning, completely disappear afterwards and new ones are formed. This is due to the fact that amino-acids, synthesised in the beginning, do not remain in the system for long, they constantly undergo decomposition and ammonification aided by the absorption of light while new ones are formed. Recently, Bahadur[10] has observed that if under sterilized conditions a mixture of paraformaldehyde, potassium nitrate, ferric chloride and water is exposed to artificial light from a 500 watt electric bulb, a number of amino-acids are synthesised in the mixture. According to him most of the amino-acids formed in the beginning disappear after sometime and new ones are formed. According to Santamaria[11]. Fucshine red helps in this synthesis.

Moreover, the foregoing data show that in the systems where phosphates are add.d, th: amount as well as the number of amino-acids increases to a greater extent in contrast to those containing very little or no phosphate. This observation further lends support to the fact that phosphates form stable complexes with proteins both in soil, plant tissue as well as *in vitro*[12], thus making protein or amino-acids molecule much more stable towards oxidation and decomposition. This observation has important bearing on the soil processes. Soils containing greater amounts of phosphate are richer in nitrogen according to Thompson[13] Biswas and Das[14], in order to study the excretion of free amino-acids in soil during the growth of a fodder legume Berseem (Trifolium alexandrium), found that soils from the plots growing Berseem were distinctly richer in their content of free amino-acids than those of the fallow soil, but what is more interesting they detected Arginine under Berseen with phosphate fertilisation.

Furthermore, a close survey of the experimental results reveals that the amino-acids produced by the slow oxidation of the said organic materials are Glycine, Alanine, Valine, Aspartic acid, Lysine, Glutamic acid, Threonine, Proline, Histidine, Arginine, Asparagine and Leucine. Amongst these, aspartic acid, glutamic acid and lysine, threonine, proline and leucine were found in abundance ; Alanine, Valine and Glycine were mostly found in the system with little or no phosphate and the rest in the systems containing phosphate in greater quantity. Of all these amino-acids Asparagine was least detected.

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Some Generating Functions for Jacobi Polynomials

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R. K. Saxena (3) has obtained a generating function for Jacobi polynomial $P_n^{\alpha,\beta}(x.)$ Here two more generating functions are obtained and interesting particular cases given.

1. Introduction

The object of the present paper is to obtain two new generating functions for the Jacobi polynomials defined by (2, p. 268)

$$(1\cdot1) \qquad P_n^{\alpha,\beta}(x) = \binom{\alpha+n}{n} {}_2F_1\left(-n,\, n+\alpha+\beta+1;\, \alpha+1;\, \frac{1-x}{2}\right)$$

where $|1-x) < 2$.

The following symbols have been used throughout this paper.

$$(a)_m = \Gamma(a+m)/\Gamma(a)$$

$$\binom{a}{b} = \frac{\Gamma(a+1)}{\Gamma(b+1)\,\Gamma(a-b+1)}$$

Also that the Appell's hypergeometric function of two variables defined by (1, p. 224)

$$(1\cdot2) \qquad F_4(\alpha,\,\beta\,;\,\gamma,\,\gamma';\,x,y) = \sum_{m,n=0}^{\infty} \frac{(a)_{m+n}\,(\beta)_{m+n}}{(\gamma)_m\,(\gamma')_n\,\underline{|m}\,\underline{|n}}\, x^m\, y^n$$

where $|x|^{\frac{1}{2}} + |y|^{\frac{1}{2}} < 1$

reduces to the form (1, p. 238)

$$(1\cdot3) \qquad F_4[a,\, \gamma+\gamma'-a-1\,;\,\gamma,\,\gamma';\,x(1-y),\,y(1-x)]$$

$$= {}_2F_1(\alpha,\,\gamma+\gamma'-\alpha-1\,;\,\gamma\,;\,x).\,{}_2F_1(a,\,\gamma+\gamma'-\alpha-1\,;\,\gamma'\,;\,y)$$

The following generating functions are developed in the next section.

$$(1\cdot4) \qquad \sum_{n=0}^{\infty} \frac{\Gamma(a+2)}{\Gamma(2n+\alpha+\beta+1)}\,(a)_n\,(b)_n\,(1+a+n)\beta\, y^{-n}\, P_n^{\alpha,\beta}(x)$$

$$\times\, F_4(a+n,\, b+n;\, \alpha+\beta+2n+2,\, 1+v\,;\, y,\, w)$$

$$= F_4\left(a, b\; ; 1+\alpha, \; 1+\nu\; ; \; \frac{y(1-x)}{2} \;, \; w \right)$$

where $|y|^{\frac{1}{2}} + |w|^{\frac{1}{2}} < 1, |1-x| < 2.$

(1·5)
$$\sum_{n=0}^{\infty} \frac{(2n+\alpha+\beta+1)\,\Gamma(\alpha+1)}{\Gamma(2\alpha+2\beta+2n+3)} \; \frac{(a)_n\,(b)_n\,(\alpha+n+1)_\beta}{c^{2n}} \; P_n^{a,\beta}(x)$$

$$\times F_4(a+n,\; b+n\; ;\; \alpha+\beta+2n+2,\; 1+\nu\; ;\; y,\;\; w$$

$$= F_4\left(a,\, b\; ;\; 1+\alpha,\; 1+\nu\; ;\; \frac{y(1-x)}{2}, w\right)$$

where $|y|^{\frac{1}{2}} + |w|^{\frac{1}{2}} < 1, |1-x| < 2$

2. Development of generating functions

To obtain the generating function (1·4) we start with Watson's formula (5, p. 140)

(2·1)
$$\sum_{n=0}^{\infty} \frac{(2n+\xi)\,\Gamma(n+\xi)}{\underline{|n}} \; {}_2F_1(n+\xi,\, -n\; ;\; \mu+1\; ;\; x^2)\, J_{2n+\xi}(z)$$

$$= \Gamma(\mu+1)\,(z/2)^{\xi-\mu}\, x^{-\mu}\, J_\mu(xz)$$

where $\xi > 0, \mu \geqslant \xi - 1, 0 < x < 1.$

On multiplying both sides of (2·1) by $z^{\lambda-1} J_\nu(\delta z)\, k_\rho(\gamma z)$ and integrating w.r.t. z in $(0, \infty)$ with the help of the integral (2, p. 373)

(2·2)
$$\int_0^\infty x^{\lambda-1} J_\mu(ax)\, \bar{J}_\nu(\beta x)\, K_\rho(\gamma x)\, dx$$

$$= \frac{2^{\lambda-2}\, a^\mu\, \beta^\nu\, \gamma^{-\lambda-\mu-\nu}\, \Gamma_{\frac{1}{2}}(\lambda+\mu+\nu-\rho)\, \Gamma_{\frac{1}{2}}(\lambda+\mu+\nu+\rho)}{\Gamma(1+\mu)\,\Gamma(1+\nu)}$$

$$\times F_4\left(\frac{\lambda+\mu+\nu-\rho}{2}\;,\; \frac{\lambda+\mu+\nu+\rho}{2}\; ;\; 1+\mu,\; 1+\nu\; ;\; -\frac{a^2}{\gamma^2},\; -\frac{\beta^2}{\gamma^2} \right)$$

where $R(\lambda+\mu+\nu) > |R(\rho)|, R(\gamma) > |I_m(a)| + |I_m(\beta)|$

we have

(2·3)
$$\sum_{n=0}^{\infty} \frac{\Gamma(n+\xi)}{\underline{|n}\,\Gamma(2n+\xi)} \; \left(\frac{\lambda+\xi+\nu-\rho}{2} \right)_n \left(\frac{\lambda+\xi+\nu+\rho}{2} \right)_n \gamma^{-2n}$$

$$\times {}_2F_1(n+\xi,\, -n\; ;\; 1+\mu\; ;\; x^2)$$

$$\times F_4\left(\frac{\lambda+\nu+\xi-\rho}{2} + n,\; \frac{\lambda+\nu+\xi+\rho}{2} + n;\; 1+\xi+2n,\; 1+\nu;\; -\frac{1}{\gamma^2},\; -\frac{\delta^2}{\gamma^2} \right)$$

[534]

$$= F_4\left(\frac{\lambda+\xi+\nu-\rho}{2}\ ,\ \frac{\lambda+\xi+\nu+\rho}{2}\ ;\ 1+\mu,\ 1+\nu\ ;\ -\frac{x^2}{\gamma^2},\ -\frac{\delta^2}{\gamma^2}\right)$$

where $\qquad R(\lambda+\xi+\nu) > |R(\rho)|,\ R(\gamma) > |I_m(\alpha)| + |I_m(\delta)|$

If we now replace $\dfrac{\lambda+\xi+\nu-\rho}{2}$ by a, $\dfrac{\lambda+\xi+\nu+\rho}{2}$ by b, x^2 by x, $-1/\gamma^2$ by y

and $-\delta^2/\gamma^2$ by w, we see that

(2·4)
$$\sum_{n=0}^{\infty} \frac{\Gamma(n+\xi)}{\underline{n}\ \Gamma(2n+\xi)}\ (a)_n\ (b)_n\ y^{-n}\ {}_2F_1(n+\xi,\ -n\ ;\ 1+\mu\ ;\ x)$$

$$\times\ F_4(a+n,\ b+n\ ;\ 1+\xi+2n,\ 1+\nu\ ;\ y,\ w)$$

$$= F_4(a,\ b\ ;\ 1+\mu,\ 1+\nu\ ;\ xy,\ w)$$

where $\qquad |y|^{\frac{1}{2}} + |w|^{\frac{1}{2}} < 1$ and $|x| < 1$

(1·4) immediately follows from (2·4) on using (1·1).

Following a similar procedure and using an integral given by Sharma (4)

(2·5)
$$\int_0^{\infty} x^{k-1}\ J_\rho(ax)\ J_\sigma(bx)\ K_\mu(cx)\ dx$$

$$= \frac{2^{k-2}\ a^\rho\ b^\sigma\ \Gamma_{\frac{1}{2}}(k+\sigma+\mu+\rho)\ \Gamma_{\frac{1}{2}}(k+\sigma-\mu+\rho)}{\Gamma(1+\rho)\ \Gamma(1+\sigma)\ c^{k+\rho+\sigma}}$$

$$\times\ F_4\left(\frac{k+\sigma-\mu+\rho}{2}\ ,\ \frac{k+\sigma+\mu+\rho}{2}\ ;\ 1+\rho,\ 1+\sigma\ ;\ -\frac{a^2}{c^2},\ \frac{b^2}{c^2}\right)$$

where $\qquad R(k+\sigma\pm\mu+\rho) > 0,\ a > 0,\ R(c-b) > 0.$

we get (1·5) on interpreting the result by (1·1).

3. Particular Cases

If we write $2y$ for y, $\nu+\alpha-a+1$ for b and $x(1-y)$ for w in (1·4) and (1·5) and make use of (1·3), we get

(3·1)
$$\sum_{n=0}^{\infty} \frac{(n+\alpha+\beta+1)\ \Gamma(\alpha+2)}{\Gamma(2n+\alpha+\beta+1)\ \Gamma(\alpha+n+1)}\ (a)_n\ (\nu+\alpha-a+1)_n\ (2y)^{-n}\ P_n^{\alpha,\beta}(x)$$

$$\times\ F_4(a+n,\ \nu+\alpha-a+n+1\ ;\ 2+\alpha+\beta+2n,\ 1+\nu\ ;\ 2y,\ x(1-y))$$

$$= {}_2F_1(a,\ \nu+\alpha-a+1\ ;\ 1+\nu\ ;\ x)\ {}_2F_1(a,\ \nu+\alpha-a+1\ ;\ 1+\alpha\ ;\ y)$$

where $\qquad |1-x| < 2,\ |y| < 1$

and

(3·2)
$$\sum_{n=0}^{\infty} \frac{(2n+\alpha+\beta+1)\ \Gamma(\alpha+\beta+n+1)\ \Gamma(\alpha+1)}{\Gamma(2\alpha+2\beta+2n+3)\ \Gamma(\alpha+n+1)}\ (a)_n\ (\alpha-a+\nu+1)_n\ c^{-2n}\ P_n^{\alpha,\beta}(x)$$

[535]

$$\times F_4(a+n, \nu+\alpha-a+1+n ; \alpha+\beta+2n+2, 1+\nu ; 2y, x(1-y))$$

$$= {}_2F_1(a, \nu+\alpha-a+1 ; 1+\nu ; x)\, {}_2F_1(a, \nu+\alpha-a+1 ; 1+\alpha ; y)$$

where $|1-x| < 2, |y| < 1$

respectively.

Lastly by virtue of the formula

(3·3)
$$P_n^{\alpha-\frac{1}{2},\ \alpha-\frac{1}{2}}(x) = \frac{(\alpha+\frac{1}{2})_n}{(2\alpha)_n}\, c_n^\alpha(x)$$

from (1·4) and (1·5) we obtain

(3·4)
$$\sum_{n=0}^{\infty} \frac{\Gamma(\alpha+\frac{3}{2})\,\Gamma(2\alpha)}{\Gamma(\alpha+\frac{1}{2})\,\Gamma(2\alpha+2n)}\,(a)_n\,(b)_n\,y^{-n}\,c_n^\alpha(x)$$

$$\times F_4(a+n, b+n ; 2\alpha+2n+1, 1+\nu ; y, w)$$

$$= F_4\left(a, b ; \alpha+\tfrac{1}{2}, 1+v ; \frac{y(1-x)}{2}, w\right)$$

where $|y|^{\frac{1}{2}} + |w|^{\frac{1}{2}} < 1, 1, |1-x| < 2$

(3·5)
$$\sum_{n=0}^{\infty} \frac{(2n+2\alpha)\,\Gamma(2\alpha)}{\Gamma(4\alpha+2n+1)}\,(a)_n\,(b)_n\,c^{-2n}\,c_n^\alpha(x)$$

$$\times F_4(a+n, b+n ; 2\alpha+2n+1, 1+\nu ; y, w)$$

$$= F_4\left(a, b ; \alpha+\tfrac{1}{2}, 1+\nu ; \frac{y(1-x)}{2}, w\right)$$

where $\qquad |y|^{\frac{1}{2}} + |w|^{\frac{1}{2}} < 1, |x| < 1$

respectively.

Acknowledgement

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Photosynthesis of Amino-acids from a mixture of nitrates and glucose or nitrates and Citric acid

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The influence of light on the formation of the green colour of Plants and its bleaching action in the dark were probably noted by Aristotle[1]. Anderson[2] observed that the amount of nitrate found in the plants would depend upon the time of the day. According to him, a plant "Solanum dulcamera" showed considerably less nitrate in the morning than later in the daytime. Oparin[3] Bernal[4] and Urey[5] have stressed the idea that life is based on organic compounds which were formed when methane, ammonia, water and hydrogen were the components of the atmosphere. Miller[6] succeeded in testing this hypothesis, producing amino-acids synthetically by circulating methane, ammonia, water and hydrogen in electric discharge.

Bahadur[7] observed that when an aqueous mixture of paraformaldehyde and potassium nitrate in the presence of Ferric chloride as a catalyst is exposed to sunlight about a dozen of amino-acids are formed. It has been found that the hydrogen ion concentration of the mixture has a great influence on the nature of the amino-acids formed. Depending upon the pH value of the mixture and the period of exposure, different amino-acids are found to be present in the mixture and what is more interesting a number of amino-acids in the beginning disappear and new ones are formed.

In the present work an attempt has been made to identify, separate and estimate the amino-acids, synthesised from a mixture of nitrates and glucose, or nitrates and Citric acid with and without dicalcium phosphate in the presence of titania as a photo-sensitizer.

Experimental

10 cc M/2 solutions of each ammonium, potassium and sodium nitrates together with 10 cc M/2 solutions of glucose or Citric acid were taken in 100 ml. Pyrex conical flasks. 0.1 gm. titania was added to each of the flasks, as photo-sensitizer. In another set of similar conical flasks, in addition to the above substance, 0.5 gm. of dicalcium phosphate was also added. Another identical set of flasks containing the above substances with and without dicalcium phosphate was also taken.

One set of flasks was exposed to light under 100 watt. electric bulb and the other set was placed beside the exposed flasks, covered with a black cloth. A thermometer was hung over the sets in order to record the temperature.

After definite intervals of time, definite portions of solutions were filtered and the filtrate was analysed for identification of amino-acids employing simple paper chromatography (Circular as well as two dimensional). Quantitative estimations were carried out colorimetrically.

Note : In the results following abbreviations have been used for amino-acids.

Glycine = Gly	Arginine = Ar.
Alanine = Al	Aspartic acid = Asp
Valine = Va	Asparagine = As
Proline = Pro	Glutamic = Glu
Histidine = His	Leucine = Leu
Lycine = Ly	Threonine = Threo

TABLE 1

10 cc M/2 Ammonium nitrate + 10 cc M/2 Glucose + 0·1 gm. Titania

Period of exposure in hours	Amino acids identified chromato-graphically	Amount of amino-acids with respect to glycine m.gm. colorimetrically	Amino-acids identified chromato-graphically	Amount of amino-acids with respect to glycine m.gm. colorimetricall
	Light		*Dark*	
8.	Ly, Al, Va, Gly	0·0475	Al, Va, Gly	Traces
16.	Ly, Al, Va, Gly	0·0907	Al, Va, Gly, Ly	0·0615
24.	Ly, Al, Va, Gly, Threo	0·1302	Al, Va, Gly, Ly	0·0926
32.	Ly, Al, Va, Gly, Threo	0·1598	Al, Va, Gly, Ly, Threo	0·1125
40.	Ly, Al, Va, Threo, Pro	0·1315	Threo, Al, Va, Gly, Ly	0·1258
48.	Pro, Ly, Va, Al, Threo	0·1103	Al, Va, Ly, Threo	0·1065

TABLE 2

10 cc M/2 Potassium nitrate + 10 cc M/2 Glucose + 0·1 gm. Titania

	Light		*Dark*	
8.	Al, Va, Gly	Traces	Al, Gly	Traces
16.	Al, Va, Gly, Ly	0·0795	Al, Gly, Va	0·0565
24.	Al, Va, Gly, Ly	0·1187	Al, Gly, Va	0·0863
32.	Al, Va, Gly, Ly, Threo	0·1455	Al, Gly, Va, Ly	0·1038
40.	Al, Va, Ly, Threo	0·1201	Al, Gly, Va, Ly	0·1152
48.	Al, Va, Ly, Threo	0·1016	Al, Va, Ly	0·0975

TABLE 3

10 cc M/2 Sodium nitrate + 10 cc M/2 Glucose + 0·1 gm. Titania

	Light		*Dark*	
8.	Al, Va, Gly	Traces	Al, Gly	Traces
16.	Al, Va, Gly	0·0705	Al, Gly	0·0457
24.	Al, Va, Gly, Ly	0·1073	Al, Gly, Va	0·0731
32.	Al, Va, Gly, Ly, Threo	0·1288	Al, Gly, Va, Ly	0·0865
40.	Al, Va, Ly, Threo	0·1092	Al, Gly, Va, Ly	0·0966
48.	Al, Va, Ly, Threo	0·0925	Al, Va, Ly	0·0835

TABLE 4

10 cc M/2 Ammonium nitrate + 10 cc M/2 Glucose + 0·5 gm. dicalcium phosphate + 0·1 gm. Titania

Period of exposure in hours	Amino-acids identified chromato-graphically	Amount of amino-acids with respect to glycine m. gm. colorimetrically	Amino-acids identified chromato-graphically	Amount of amino-acids with respect to glycine m.gm. colorimetrically
	Light		*Dark*	
8.	Al, Va, Gly, Ly, Threo	0·0556	Al, Va, Gly, Ly	0·0405
16.	Al, Va, Ly, Threo, Pro	0·1052	Al, Va, Gly, Ly	0·0797
24.	Al, Va, Ly, Threo, Pro Glu	0·1471	Al, Va, Ly, Threo, Pro	0·1093
32.	Al, Va, Ly, Threo, Pro, Glu, Asp	0·1795	Al, Va,Ly, Glu, Threo, Pro	0·1308
40.	Al, Va, Ly, Threo, Pro, Glu, Asp	0·2096	Al, Va,Ly, Threo, Pro, Glu	0·1495
48.	Va, Ly, Threo, Pro, Glu, Asp	0·1835	Al, Va, Ly, Threo, Pro, Glu	0·1319

TABLE 5

10 cc M/2 Potassium nitrate + 10 cc M/2 Glucose + 0·5 gm. Dicalcium phosphate + 0·1 gm. Titania

	Light		*Dark*	
8.	Al, Va, Gly, Ly	0·0515	Al, Va, Gly	Traces
16.	Al, Va, Gly, Ly, Threo	0·0998	Al, Gly, Va, Ly	0·0718
24	Al, Va, Ly, Threo, Pro	0·1397	Al, Va, Gly, Ly, Threo	0·0991
32.	Al, Va, Ly, Threo, Pro, Glu	0·1685	Al, Va, Gly, Ly, Pro, Threo	0·1194
40.	Al, Va, Ly, Threo, Pro, Gly	0·1956	Gly, Al, Va, Ly, Threo, Pro	0·1359
48.	Va, Ly, Threo, Pro, Glu	0·1757	Al, Va, Ly, Threo, Pro	0·1205

TABLE 6

10 cc M/2 Sodium nitrate + 10 cc M/2 Glucose + 0·5 gm. Dicalcium phosphate + 0·1 gm. Titania

	Light		*Dark*	
8.	Al, Va, Gly	Taaces	Al, Gly	Traces
16.	Al, Va, Gly, Ly	0·0775	Al, Gly, Va	0 0695
24.	Al, Va, Gly, Ly, Threo	0·1127	Al, Gly, Va, Ly	0·0955
32.	Al, Va, Ly, Threo, Pro, Glu	0·1406	Al, Gly, Va, Ly, Threo, Pro	0·1143
40.	Va, Ly, Threo, Pro, Glu	0·1654	Gly. Al, Va, Ly, Threo, Pro	0·1295
48.	Va, Ly, Threo, Pro, Glu	0·1463	Al, Va, Ly, Threo, Pro	0·1156

TABLE 7
10 cc M/2 Ammonium nitrate + 10 cc M/2 Citric acid + 1 gm. Titania

Period of exposure in hours	Amino-acids identified chromato-graphically	Amount of amino-acids with respect to glycine m. gm. colorimetrically	Amino-acids identified chromato-graphically	Amount of amino-acids with respect to glycine m. gm. colorimetrically
	Light		*Dark*	
8.	Gly, Al, Va,	0·0453	Al, Gly	Traces
16.	Gly, Al, Va, Ly	0·0885	Gly, Al, Va	0·0504
24.	Al, Va, Gly, Ly, Threo	0·1269	Gly, Al, Va, Ly	0·0803
32.	Al, Va, Gly, Ly, Threo, Pro,	0·1554	Gly, Al, Va, Ly, Threo	0·0992
40.	Al, Va, Ly, Threo, Pro	0·1275	Gly, Al, Va, Ly, Threo	0·1119
48.	Al, Va, Ly, Threo, Pro	0·1076	Al, Va, Gly, Threo	0·0928

TABLE 8
10 cc M/2 Potassium nitrate + 10 cc M/2 Citric acid + 0·1 gm. Titania

Period of exposure in hours	*Light*		*Dark*	
8.	Al, Va, Gly	Traces	Al, Gly	Traces
16.	Al, Va, Gly	0·0711	Al, Gly, Va	0·0449
24.	Al, Va, Gly, Ly,	0·1094	Al, Gly, Va	0·0732
32.	Al, Va, Gly, Ly, Threo	0·1348	Al, Gly, Va, Ly	0·0903
40.	Al, Va, Ly, Threo	0·1102	Al, Gly, Va, Ly	0·1014
48.	Al, Va, Ly, Threo	0·0927	Al, Va, Ly	0·0835

TABLE 9
10 cc M/2 Sodium nitrate + 10 cc M/2 Citric acid + 0·1 gm. Titania

Period of exposure in hours	*Light*		*Dark*	
8.	Al, Gly	Traces	Al, Gly	Traces
16.	Al, Gly, Va	0·0625	Al, Gly	Traces
24.	Al, Gly, Va, Ly,	0·0979	Al, Gly, Va	0·0683
32.	Al, Gly, Va, Ly, Threo	0·1188	Al, Gly, Va, Ly,	0·0811
40.	Al, Gly, Va, Ly, Threo	0·0997	Al, Gly, Va, Ly	0·0901
48.	Al, Va, Ly, Threo	0·0843	Al, Va, Ly,	0·0749

TABLE 10
10 cc M/2 Ammonium nitrate + 10 cc M/2 Citric acid + 0·5 gm. Dicalcium phosphate + 0·1 gm. Titania

Period of exposure in hours	*Light*		*Dark*	
8.	Al, Gly, Ly	0·0495	Al, Va, Gly	Traces
16.	Al, Va, Gly, Ly, Threo	0·0977	Al, Va, Gly	0·0655
24.	Al, Va, Gly, Ly, Threo, Pro	0·1378	Al, Va, Gly, Ly, Threo	0·0938
32.	Al, Va, Ly, Threo, Glu, Pro	0·1689	Al, Va, Gly, Ly, Threo, Pro	0·1139
40.	Al, Va, Ly, Threo, Glu, Pro	0·1914	Gly, Al, Va, Ly, Threo, Pro	0·1312
48.	Al, Va, Ly, Threo, Glu, Pro	0·1716	Al, Va, Ly, Threo, Pro	0·1151

TABLE 11

10 cc M/2 Potassium nitrate + 10 cc M/2 Citric acid + 0·5 gm. Dicalcium phosphate + 0·1 gm. Titania

Period of exposure in hours	Amino-acids identified chromato-graphically	Amount of amino-acids with respect to glycine m. gm. colorimetrically	Amino-acids identified chromato-graphically	Amount of amino-acids with respect to glycine m. gm. colorimetrically
	Light		*Dark*	
8.	Al, Va, Gly	Traces	Al, Gly	Traces
16.	Al, Va, Gly, Ly	0·0857	Al, Gly, Va	0·0605
24.	Al, Va, Gly, Ly, Threo	0·1229	Al, Gly, Va, Ly	0·0866
32.	Al, Va, Gly, Ly, Threo, Pro	0·1508	Al, Gly, Va, Ly, Threo	0·1058
40.	Al, Va, Ly, Threo, Pro	0·1724	Gly, Al, Va, Ly, Threo	0·1214
48.	Al, Va, Ly, Threo, Pro	0·1535	Al, Va, Ly, Threo	0·1071

TABLE 12

10 cc M/2 Sodium nitrate + 10 cc M/2 Citric acid + 0·5 gm. Dicalcium phosphate + 0·1 gm. Titania

	Light		*Dark*	
8.	Al, Va, Gly	Traces	Al, Gly	Traces
16.	Al, Va, Gly, Ly	0·0706	Al, Gly, Va	0·0615
24.	Al, Va, Gly, Ly, Threo	0·1041	Al, Gly, Va, Ly	0·0868
32.	Al, Va, Gly, Ly, Threo, Pro	0·1305	Gly, Al, Va, Ly, Threo	0·1049
40.	Al, Va, Ly, Threo, Pro	0·1514	Gly, Al, Va, Ly, Threo	0·1197
48.	Al, Va, Ly, Threo, Pro	0·1339	Al, Va, Ly, Threo	0·1065

Discussion

From the experimental results (*vide* Tables 1–12) it can be observed that when nitrate solutions mixed with glucose or citric acid as a source of carbon in the presence of titania as photocatalyst are exposed to light, amino acids are synthesised in smaller amounts. It is also observed that the number and amount of amino-acids synthesised is greater in the sets exposed to light than those kept in the dark showing thereby the influence of light on amino-acid synthesis.

Moreover our experimental results show that the number and also the amount of amino-acids synthesised is slightly greater in the sets containing glucose that those containing citric acid as a source of carbon. It seems that glucose acts as a better energy material than citric acid in the system resulting in the formation of amino-acids.

It is further observed that in the system containing nitrate solutions, titania and glucose or citric as carbonaceous material, the amount of amino-acids gradual-ly increases but after 32 hours of exposure, the amount of amio-acid tends to decrease steadily. The amino-acids synthesised undergo ammonification and decomposition constantly aided by the absorption of light radiations, thus resulting in the loss of amino-acids on prolonging the period of exposure. In the sets kept

in the dark, the amount of amino-acids tends to decrease after 40 hours showing thereby that the ammonification and decomposition of amino-acid is slower than in similar sets exposed to light. These observations seem to be in agreement with the experimental results of Dhar and Mukerji[8] who reported the disappearance of amino-acid photosynthesised, on prolonging the period of exposure.

Our experimental results further indicate that some of the amino-acid formed in the beginning disappear at a later stage while new ones are formed. The amino-acid synthesised in the beginning might have undergone oxidation aided by the absorption of light and new molecules were formed on prolonging the period of exposure as has been observed by Ranganayaki and Bahadur[9] in their experiments with Paraformaldehyde and nitrate solutions.

Furthermore, it is observed that the number and the amount of amino-acids synthesised varies with different nitrate solutions in the following order :

$$NH_4NO_3 > KNO_3 > NaNO_3$$
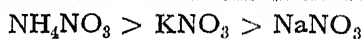
It is interesting to note that solutions containing dicalcium phosphate produce greater yield of amino-acid than the solutions containing no phosphate. Moreover the loss of amino-acids is also checked in the solutions containing phosphate as the amino-acid content tends to decrease after 40 hours of exposure in contrast to the sets containing no phosphate where the loss of amino-acid synthesised starts after 32 hours of exposure. These observations are important from the view point that phosphates form stable complexes with proteins in soil, plant tissue and in Vitro[10], in presence of phosphate which make the protein or amino-acid molecule more stable towards oxidation and decomposition. Moreover, it appears that the amino-acids synthesised do not remain in the solution for long. They constantly undergo decomposition and ammonification aided by light absorption while new ones are formed, addition of phosphates checks the decomposition of these amino-acids by forming stable phosphorylated compounds with them, with the result that the total amount of amino-acids detected colorimetrically appears to be greater in solution containing dicalcium phosphate.

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On a class of periodic orbits in the restricted problem of three bodies in a three dimensional coordinate system

By

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Abstract

In this paper a periodic solution of the restricted problem of three bodies is shown to exist. Here we have taken the solution of the plane problem of two fixed centres for the generating solution.

1. Equation of motion and their transformations

We shall consider here the circular restricted problem of three bodies in a three-dimensional coordinate system. Let us take two finite masses m_1 and m_2 placed at the points M_1 and M_2 respectively. We shall take them to be point-masses. Let the centre of inertia of the two masses m_1 and m_2 be taken for the origin. The two masses m_1 and m_2 will be assumed to rotate uniformly about the origin with the angular velocity n. Let the orbital plane of these point-masses be taken for the xy–plane and the line joining M_1 and M_2 for the x-axis. Let the distance $M_1 M_2$ be equal to $2c$. Then the force function U of the point–masses at $M(x, y, z)$, whose motion is under consideration, is

$$U = f\left(\frac{m_1}{r_1}, + \frac{m_2}{r_2}\right)$$

where

$$r_1{}^2 = M_1 M^2 = \left(x - \frac{2m_2 c}{m_1 + m_2}\right)^2 + y^2 + z^2$$

$$r_2{}^2 = M_2 M^2 = \left(x + \frac{2m_1 c}{m_1 + m_2}\right)^2 + y^2 + z^2$$

and the kinetic energy is given by $2T = (\dot{x} - ny)^2 + (\dot{y} + nx)^2 + \dot{z}^2$

For the generalized coordinates, we take $x = Q_1, y = Q_2, z = Q_3$ and so for the impulses, we have

$$P_1 = \dot{Q}_1 - nQ_2, P_2 = \dot{Q}_2 + nQ_1, P_3 = \dot{Q}_3$$

and for the Hamiltonian function,

$$H = \tfrac{1}{2}(P_1{}^2 + P_2{}^2 + P_3{}^2) + n(P_1 Q_2 - P_2 Q_1) - U(Q_1, Q_2, Q_3)$$

Introducing the contact transformation

[543]

$$\xi_1 = Q_1 + \frac{m_1 - m_2}{m_1 + m_2} c, \ \xi_2 = Q_2, \ \xi_3 = Q_3$$
$$\eta_1 = P_1 \qquad\qquad , \ \eta_2 = P_2, \ \eta_3 = P_3$$

we find that the Hamiltonian function reduces to

$$H = \tfrac{1}{2}(\eta_1{}^2 + \eta_2{}^2 + \eta_3{}^2) + n(\eta_1\,\xi_2 - \eta_2\,\xi_1) + n\,\frac{m_1 - m_2}{m_1 + m_2} c\eta_2 - $$
$$- U(\xi_1, \xi_2, \xi_3) \tag{1}$$

Taking into consideration our further study, we shall prefer to use the following elliptic canonic variables $(q_1, q_2, q_3 \ ; p_1, p_2, p_3)$ defined by formulae

$$\xi_1 = c \cos q_1 \operatorname{ch} q_2, \ \eta_1 = \frac{1}{cI}[-p_1 \sin q_1 \operatorname{ch} q_2 + p_2 \cos q_1 \operatorname{sh} q_2]$$

$$\xi_2 = -c \sin q_1 \operatorname{sh} q_2 \cos q_3, \ \eta_2 = \frac{1}{cI}[-p \cos q_1 \operatorname{sh} q_2 \cos q_3 - p_2 \sin q_1 \operatorname{ch} q_2 \cos q_3 +$$

$$+ p_3 \frac{I \sin q_3}{\sin q_1 \operatorname{sh} q_2}]$$

$$\xi_3 = -c \sin q_1 \operatorname{sh} q_2 \sin q_3, \ \eta_3 = \frac{1}{cI}\left[-p \cos q_1 \operatorname{sh} q_2 \sin q_3 - p_2 \sin q_1 \operatorname{ch} q_2 \sin q_3 - \right.$$

$$\left. - p_3 \frac{I \cos q_3}{\sin q_1 \operatorname{sh} q_2}\right]$$

where $I = \operatorname{ch}^2 q_2 - \cos^2 q_1$.

It is easily seen that the Hamilton function (1) now reduces to

$$H = \frac{1}{2c^2 I}\left(p_1{}^2 + p_2{}^2 + p_3{}^2\,\frac{I}{\sin^2 q_1 \operatorname{sh}^2 q_2}\right) - \frac{f}{cI}[(m_1 + m_2)\operatorname{ch} q_2 + (m_1 - m_2)\cos q_1] +$$

$$+ \frac{n}{I}\left[p_1 \operatorname{sh} q_2 \cos q_3\left(\operatorname{ch} q_2 - \frac{m_1 - m_2}{m_1 + m_2}\cos q_1\right) + \right.$$

$$+ p_2 \sin q_1 \cos q_3\left(\cos q_1 - \frac{m_1 - m_2}{m_1 + m_2}\operatorname{ch} q_2\right) -$$

$$\left. - p_3 I \sin q_3\left(\cos q_1 \operatorname{ch} q_2 - \frac{m_1 - m_2}{m_1 + m_2}\right)\sin q_1 \operatorname{sh} q_2\right]$$

Then the differential equations of motion in the canonical form can be written as

$$\frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}, \ \frac{dp_i}{dt} = -\frac{\partial H}{\partial q_i} \ (i = 1, 2, 3) \tag{2}$$

2. Regularisation of the equations of motion

A look at the expression for H shows that at the instant of collision with any of the two finite masses, the right hand sides of (2) reduce to infinity. Thus in the neighbourhoods of the two finite masses the solution seems to be singular.

In order to have a regular solution, we shall try to regularize our differential equations. For this we shall introduce an independent variable τ instead of t by means of the relation $dt = I\, d\tau$

Since the Hamiltonian function H does not involve t explicitly, so the system of equations (2) will admit the energy integral

$$H = h \tag{3}$$

With the introduction of τ our equations (2) will be transformed into

$$dq_i/d\tau = I\, \partial H/\partial p_i, \quad dp_i/d\tau = -I\, \partial H/\partial q_i \tag{4}$$

We find that $\dfrac{\partial}{\partial p_i}(H I) = I\dfrac{\partial H}{\partial p_i} + H\dfrac{\partial I}{\partial p_i}$, whence by using the integral

we have $I\dfrac{\partial H}{\partial p_i} = \dfrac{\partial\{(H-h)\,I\}}{\partial p_i}$ and similarly, $I\dfrac{\partial H}{\partial q_i} = \dfrac{\partial\{(H-h)\,I\}}{\partial q_i}$

Consequently, (4) may be written as

$$\frac{dq_i}{d\tau} = \frac{\partial \Omega}{\partial p_i} , \quad \frac{dp_i}{d\tau} = -\frac{\partial \Omega}{\partial q_i} \tag{5}$$

where $\Omega = (H - h)\, I$.

Here the integral of energy may be written as $\Omega = 0$.

Now as in the equations (5), r_1 or r_2 no longer appears in the dominators and so equations (5) may be taken to be regularised.

3. Generating solution

With (Charlier, 1907) we shall decompose the characteristic function Ω into two parts given as $\Omega = \Omega_0 + n\,\Omega_1$

where

$$\Omega_0 = \frac{1}{2c}\left[p_1^2 + p_2^2 + p_3^2\, \frac{I}{\sin^2 q_1\, \mathrm{sh}^2 q_2} \right] -$$

$$-\frac{f}{c}\left[(m_1 + m_2)\,\mathrm{ch}\,q_2 + (m_1 - m_2)\cos q_1 \right] - h(\mathrm{ch}^2 q_2 - \cos^2 q_1)$$

$$\Omega_1 = p_1\left[\mathrm{ch}\,q_2 - \frac{m_1 - m_2}{m_1 + m_2}\cos q_1 \right]\mathrm{sh}\,q_2 \cos q_3 +$$

$$+ p_2\left[\cos q_1 - \frac{m_1 - m_2}{m_1 + m_2}\,\mathrm{ch}\,q_2 \right]\sin q_1 \cos q_3 -$$

$$- p_3\left[\cos q_1\, \mathrm{ch}\,q_2 - \frac{m_1 - m_2}{m_1 + m_2} \right]\frac{I \sin q_3}{\sin q_1\, \mathrm{sh}\,q_2} ,$$

We shall take the solution of the equations (5) corresponding to the Hamiltonian function Ω_0 for the generating solution. This generating solution is seen to be of the form [Demin, 1960]:

$$\cos q_{10} = -\frac{a + cn\,[\sigma_0\,(\tau - \tau_0), k]}{1 + a\, cn\,[\sigma_0\,(\tau - \tau_0), k]}$$

$$p_{10} = - \frac{c^2 \, \sigma_0 \, \sqrt{1 - a^2} \; dn \, [\, \sigma_0 \, (\tau - \tau_0), k \,]}{1 + a \; cn \, [\, \sigma_0 \, (\tau - \tau_0), k \,]} \tag{6}$$

$$ch \; q_{20} = - f \, \frac{m_1 + m_2}{2 h \, c}$$

$$p_{20} = 0 = q_{30} = p_{30} \, ,$$

[The suffix (0) denotes that the value has been obtained under the assumption that $n = 0$]

$$\text{Here } a = \frac{m_1 + m_2}{4 (m_1 - m_2) \; ch \; q_{20}} \left[\sqrt{1 + 2 \, \frac{m_1 - m_2}{m_1 + m_2} \; ch \; q_{20} + ch^2 \; q_{20}} \, - \right.$$

$$\left. - \sqrt{1 - 2 \, \frac{m_1 - m_2}{m_1 + m_2} \; ch \; q_{20} + ch^2 \; q_{20}} \; \right]^2$$

$$k^2 = \tfrac{1}{2} \left[1 - \frac{(m_1 + m_2) \; sh^2 \; q_{20}}{\sqrt{(m_1 + m_2)^2 \; sh^4 \; q_{20} + 16 \, m_1 \, m_2 \; ch \; q_{20}}} \right]$$

$$\sigma_0^2 = \frac{f}{c^3 \; ch \; q_{20}} \, \sqrt{(m_1 + m_2)^2 \; sh^4 \; q_{20} + 16 \, m_1 \, m_2 \; ch^2 \; q_{20}}$$

In the solutions (6) h and τ_0 are arbitrary constants of integration. It is clear that the generating solution (6) define a family of elliptic orbits with the focii at the attracting masses. This solution is periodic relative to τ with the period T where

$$T = \frac{4}{\sigma_0} \int_0^{\pi/2} \frac{d\phi}{\sqrt{1 - k^2 \sin^2\phi}} = \frac{4}{\sigma_0} \, K \, (k)$$

4. First Approximation of the general solution

We shall now aim to find the solution in series expanded in ascending powers of n, i.e.

$$\left. \begin{array}{l} q_1 = q_{10} + \sum\limits_{i=1}^{\infty} n^i \, q_{1i}, \; q_2 = q_{20} + \sum\limits_{i=1}^{\infty} n^i \, q_{2i}, \; q_3 = q_{30} + \sum\limits_{i=1}^{\infty} n^i \, q_{3i} \\[3mm] p_1 = p_{10} + \sum\limits_{i=1}^{\infty} n^i \, p_{1i}, \; p_2 = p_{20} + \sum\limits_{i=1}^{\infty} n^i \, p_{2i}, \; p_3 = p_{30} + \sum\limits_{i=1}^{\infty} n_i \, p_{3i} \end{array} \right\} \tag{7}$$

In this section we shall try to find q_{11}, q_{21}, q_{31}, p_{11}, p_{21}, and p_{31}. For this evaluation we shall substitute the series (7) for $q_1, q_2, \ldots$ in the differential equations (5) and we shall equate the coefficients of n on the two sides. Thus we shall have the following system of differential equations for q_{11}, q_{21},

$$\frac{dq_{ij}}{d\tau} = p_{i1} \left(\frac{\partial^2 \Omega_0}{\partial p_i^2} \right)_0 + \left(\frac{\partial \Omega_1}{\partial p_i} \right)_0 \tag{8}$$

$$\frac{dp_{i1}}{\partial t} = - q_{i1} \left(\frac{\partial^2 \Omega_0}{\partial q_i^2} \right)_0 - \left(\frac{\partial \Omega_1}{\partial q_i} \right)_0 \tag{9}$$

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The index zero in the equations (8) and (9) denotes that in the partial derivatives of Ω_0 and Ω_1 with respect to q_i and p_i we should replace q_i and p_i by q_{i0} and p_{i0} after the differentiation. Equations (8) and (9) may be seen to be reduced to the following form :

$$\frac{d^2 q_{11}}{d\tau^2} + \left[\frac{f}{c^3} (m_1 - m_2) \cos q_{10} + \frac{f}{c_3} (m_1 + m_2) \frac{\cos 2 q_{10}}{\text{ch } q_{20}} \right] q_{11} = 0 \qquad (10)$$

$$\frac{d^2 q_{21}}{d\tau^2} + \frac{f(m_1 + m_2)}{c^3 \text{ ch } q_{20}} \text{ sh}^2 q_{20} q_{21} = - (\text{ch } 2 q_{20} - \cos 2 q_{10}) \frac{p_{10}}{c^2} \qquad (11)$$

$$\frac{dq_{31}}{d\tau} = \frac{p_{31}}{c^2} \frac{I}{\sin^2 q_{10} \text{ sh}^2 q_{20}} \qquad (12)$$

$$\frac{dp_{31}}{d\tau} = 0 \qquad (13)$$

With [Demin, 1960] the solution of the differential equations (10) - (13) may be written as

$$q_{11} = q_{10}' \left(\beta_3 \int \frac{d\tau}{q_{10}'^2} + \beta_4 \right) \qquad (14)$$

$$p_{11} = c^2 q_{10}'' \left(\beta_3 \int \frac{d\tau}{q_{10}'^2} + \beta_4 \right) + \frac{c^2 \beta_3}{q_{10}'} \qquad (15)$$

$$q_{21} = \beta_1 \cos \sigma(\tau - \tau_0) + \beta_2 \sin \sigma(\tau - \tau_0) + F(\tau) \qquad (16)$$

$$p_{21} = - \beta_1 \sigma c^2 \sin \sigma(\tau - \tau_0) + \beta_2 \sigma c^2 \cos \sigma(\tau - \tau_0) + F_1(\tau) \qquad (17)$$

$$q_{31} = \frac{\beta_5}{c^2} \left[\tau(\text{cosech}^2 q_{20} + 2) - \frac{2 - a^2}{\sigma_0(1 - a^2)} \{ E(u) + \sin^{-1} (cn \, u \, dn \, u) \} - \right.$$

$$\left. - \frac{2 a}{\sigma_0(1 - a^2)} \frac{dn \, u}{sn \, u} \right] + \beta_6 \qquad (18)$$

$$p_{31} = \beta_5 \qquad (19)$$

where

$$\int \frac{d\tau}{q_{10}'^2} = \frac{\pi^2 \tau}{4 \sigma_0^2 (1 - a^2) K} \left[\frac{1}{k'^2} \left(1 + 2 \sum_{\nu=1}^{\infty} \text{sech}^2 2\nu\rho \right) + \right.$$

$$\frac{2a^2}{k^2} \sum_{\nu=1}^{\infty} \text{cosech}^2 (2 \nu - 1) \rho + P. T.$$

$$K'^2 = 1 - K^2, \rho = \frac{\pi K'}{2K}, K' = K (k')$$

$$P. T = \text{periodic terms in } \tau$$

$$\frac{1}{q'_{10}} = - \frac{1 + a \, cn \, [\sigma_0 (\tau - \tau_0), k]}{\sigma_0 \sqrt{1 - a^2} \, dn \, [\sigma_0 (\tau - \tau_0), k]}$$

$$F (\tau) = \text{a periodic function in } \tau$$

$$F_1 (\tau) = \text{a periodic function in } \tau.$$

The solutions (14) – (19) are the first approximations of the general solution. In the same way the second and the higher approximations may be calculated.

5. Existence of periodic solution

In this section we shall examine if there exists a periodic solution with the period of the generating solution, i e., T. It is clear that if q_{11}, q_{21}, q_{31}, p_{21} and p_{31} are the solutions and if they are periodic with the period T, then the condition of periodicity may be written as

$$\psi_1 = q_{11}(\tau_0 + T) - q_{11}(\tau_0) = 2s_1\pi$$
$$\psi_2 = q_{21}(\tau_0 + T) - q_{21}(\tau_0) = 0$$
$$\psi_3 = q_{31}(\tau_0 + T) - q_{31}(\tau_0) = 2 s_2\pi$$
$$\psi_4 = p_{11}(\tau_0 + T) - p_{11}(\tau_0) = 0$$
$$\psi_5 = p_{21}(\tau_0 + T) - p_{21}(\tau_0) = 0$$
$$\psi_6 = p_{31}(\tau_0 + T) - p_{31}(\tau_0) = 0$$

where s_1 and s_2 are arbitrary integers. By virtue of the solutions (14) – (19), we have

$$\psi_1 = \beta_3 \frac{\pi^2}{\sigma_0^2 (1 + a) \sqrt{1 - a^2}\, K}\left[\frac{1}{k'^2}\left(1 + 2 \sum_{\nu=1}^{\infty} \operatorname{sech}^2 2\nu\rho\right) + \right.$$

$$\left. + \frac{2a^2}{k^2} \sum_{\nu=1}^{\infty} \operatorname{cosech}^2 (2\nu - 1)\rho\right] = 2 s_1 \pi$$

$$\psi_2 = \beta_1 (\cos \sigma T - 1) + \beta_2 \sin \sigma T = 0$$

$$\psi_3 = \frac{\ell_5}{c^2}\left\{(\operatorname{cosech}^2 q_{20} + 2) - (1 - k^2)\frac{2 - a^2}{1 - a^2}\right\} T = 2 s_2 \pi \qquad (20)$$

$$\psi_4 = 0$$

$$\psi_5 = -\beta_1 \sigma c^2 \sin \sigma T + \beta_2 \sigma c^2 (\cos \sigma T - 1) = 0$$

$$\psi_6 = 0$$

Now if the constants β_1, β_2, β_3 and ℓ_5 can be found such as to satisfy the equations (2), then the periodic solution will exist. We have an integral of energy and so one relation can be taken to be dependent, say, $\psi_4 = 0$. As we are now left with four constants β_1, β_2, ℓ_3 and β_5 and we have five equations, so if it is shown that a fundamental determinant of the fourth order is not zero, then the existence of the periodic solution will be proved. We find now that

$$\frac{\partial(\psi_1, \psi_2, \psi_3, \psi_5)}{\partial(\beta_1, \beta_2, \beta_3, \beta_5)} = 4 \sigma \sin^2\frac{\sigma\tau}{2}\left\{\operatorname{cosech}^2 q_{20} + (1 - k^2)\frac{2 - a^2}{1 - a^2}\right\} T \times$$

$$\times \frac{\pi^2}{\sigma_0^2 (1 + a)\sqrt{1 - a^2}\, K}\left[\frac{1}{k'^2}\left(1 + 2 \sum_{\nu=1}^{\infty} \operatorname{sech}^2 2\nu\rho\right) + \right.$$

$$\left. + \frac{2a^2}{k^2} \sum_{\nu=1}^{\infty} \operatorname{cosech}^2 (2\nu - 1)\rho\right]$$

is definitely distinct from zero, if $2k\sigma \neq p\sigma_0 \pi$ where p is an arbitrary integer. We can proceed for the existence of periodic solution in the further approximations similarly, but in that case the question of convergence of the series representing the solution will arise.

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The Effect of Mode of Micronutrient Application on Wheat : Yield, Uptake of Nitrogen and Phosphorus, and the relationship between Copper, Nitrogen, Zinc and Phosphorus

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Abstract

Foliar spray of copper or zinc improved the yield of grains and straw at Pant Nagar. Copper, zinc and all micro-nutrients in combination when applied to soil or foliage only gave beneficial effect at Patiala.

All the applied trace elements were effective in improving wheat straw nitrogen at Pant Nagar. Foliar fertilization of Zn, B, Cu and Mn, and soil dressing with Mn, Mo and Cu had a beneficial effect on grain nitrogen. At Patiala, soil application of Mo alone, increased grain nitrogen. The phosphorus content of the straw and of grains, improved under slightly acidic soils of Pant Nagar only.

The possibility of a relationship between the amounts of zinc, phosphorus, copper and nitrogen, present in straw and grain from both the places was also studied.

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Micronutrients play a significant role in the plant metabolism and growth, hence their application not only affects the yield but the quality of the crop produce also. There are conflicting reports with regards to the effect of trace element application on plant nitrogen and phosphorus. Application of Cu, Mn, Zn, B and Mo increased nitrogen content of plants (Ozolina[21], Kokin[18], Sadaphal and Das[24], Aliev[1], Kholi[16], and Gautam et al[11]). The same authors have shown that fertilization with the same trace elements decreased N content as well. Similar to the effect observed on the uptake of nitrogen, the addition of trace elements may offset the plant phosphorus in either way (Kastori and Saric[15], Gopalkrishnan[12] Fedorenko[6], and Karim and Deraj[14]).

Investigations have been made to find out the relationship between the uptake of Cu, N, Zn and P. Positive interaction between plant N and Cu has been reported in citrus seedlings (Vsevolozhskaya[27]), barley plants (Ozolina[21]) and in cholum and ragi (Gopalkrishnan[12]). For Zn and P, positive correlation was found in rye and cauliflower (Luders[20]), citrus (Frank[19]), and grapes (Fedorenko[6]). Inverse relationship between Zn and P has been also reported (Thompson[26]) Paribok et al[22] and Terman et al[25]).

The present study aims at finding out the effect of Cu, Zn, Mo, Mn and B, applied to the soil as well as to the foliage of two high yielding varieties of wheat on yield, uptake of nitrogen and phosphorus and on possible relation of Cu, N, Zn, and P contents.

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Experimental

The soil and plant samples for the present investigations were collected at harvesting stage from Model Agronomic Trial Centres at Patiala and Pant Nagar. The soils of both these places were sandy loam in texture. Their chemical analysis is reported in table 1A. Wheat varieties PV-18 and C-306 were grown at Patiala and Pant Nagar respectively. The experiments were laid in a randomized block design, giving similar treatments at both the places (table 1B). Each treatment was replicated four times. Net plot size at Pant Nagar was 32 square metres and at Patiala it was 25 square metres. The application of the fertilizer doses was made as given in tabels 1C and 1D. Two foliar applications of trace elements were made after 40 and 55 days of sowing. Soil fertilization of NPK and trace elements was done before sowing (broadcasting).

TABLE 1A
Soil Chemical Analysis

Analysed	Pant Nagar	Patiala
Organic Carbon%	0·89	0·39
Total Nitrogen%	0·16	0·075
CaCO₃%	1·64	1·43
pH	6·2	7·5

TABLE 1B
Treatments at Patiala and Pant Nagar

S. No.	Treatment	Symbol	Mode of application
1.	Control	C	—
2.	NPK	NPK	Soil application
3.	NPK + Cu	NPK + Cu(S)	Soil application
4.	NPK + Zn	NPK + Zn(S)	Soil application
5.	NPK + B	NPK + B (S)	Soil application
6.	NPK + Mn	NPK + Mn (S)	Soil application
7.	NPK + Mo	NPK + Mo (S)	Soil application
8.	NPK + All trace elements	NPK + All (S)	Soil application
9.	Spartin	Sp (S)	Soil application
10.	NPK + Cu	NPK + Cu (F)	Foliar spray
11.	NPK + Zn	NPK + Zn (F)	Foliar spray
12.	NPK + B	NPK + B (F)	Foliar spray
13.	NPK + Mn	NPK + Mn (F)	Foliar spray
14.	NPK + Mo	NPK + Mo (F)	Foliar spray
15.	NPK + All trace elements	NPK + All (F)	Foliar spray

Basal doses and forms of NPK fertilizers applied in kgs per hectare

Name of the centre	Rate of application		
	N	P	K
Patiala	120 (C. A. N)	60 (Superphos.)	60 (KCl)
Pant Nagar	35 (Urea)	35 (Superphos..)	35 (KCl)

TABLE 1D

Dosage and form of trace element applied in kgs per hectare

Microelement	Soil application	Foliar application	Name of the compound used
Manganese	12·33	6·16	Mn SO$_4$·4H$_2$O
Zinc	5·70	2·85	ZnSO$_4$·7H$_2$O
Copper	6·36	3·18	CuSO$_4$·5H$_2$O
Boron	1·15	0·55	Na$_2$B$_4$O$_7$·10H$_2$O
Molybdenum	0·46	0·23	Na$_2$MoO$_4$
Mixture of all trace elements	370·00	–	*Spartin-B

*Spartin is supposed to contain all the essential trace elements along with NPK.

The soil samples representative of a particular plot were collected at the harvesting stage. Only surface samples from 0·22 cms were taken with the help of a soil sample and packed in alkathene bags. After drying in the air, the soil samples were powdered in a porcelain pestle and mortar. Before use, the porcelain pestle and mortar was treated with hydrochloric acid, washed with double distilled water and cleaned with a piece of cloth. The powdered sample was passed through a muslin cloth sieve and stared in an alkathene bag.

The representative samples of the plant material, collected at the harvesting stage, were dried in an oven at 100°C. The plant samples were further powdered, screened and stored in a manner already detailed for the preparation of soil samples. The results of chemical analysis of composite soil and plant samples were expressed on oven dry basis at 100°C.

Soil pH was determined in soil water suspension of 1 : 2·5 ratio with the help of Beckman glass electrode pH meter. Organic carbon and calcium carbonate were estimated as mentioned in Piper[23]. The results of organic carbon were expressed as Walkley and Black values. Kjeldahl's method was followed for the determination of total nitrogen in soils (A. O. A. C.[3]).

Total nitrogen of the plant samples were determined by the Kjeldahl-Gunning's method as given in A. O. A. C.[3]. Total phosphorus, zinc and copper were estimated in the triacid (60% HClO$_4$, conc. H$_2$SO$_4$ and conc. HNO$_3$ in 2 : 1 : 5 by volume) extract. Phosphorus was determined by the chlorimetric method of Fiske and Subbarow[7] as modified by King[17]. Zinc and copper were estimated using dithizone (A. O. A. C.[3]) and carbamate (Chang and Bray[5]) reagents.

TABLE 2

Effect of trace element application on the yield of wheat

S. No.	Treatment symbol	Yield in Kgs per plot			
		Patiala		Pant Nagar	
		Grain	Straw	Grain	Straw
1.	C	7·00	11·97	10·25	17·70
2.	NPK	12·02	23·97	10·80	20·70
3.	NPK + Cu (S)	13·07	24·92	10·15	15·35
4.	NPK + Zn (S)	12·97	24·27	11·50	18·10
5.	NPK + B (S)	12·10	25·15	10·65	17·20
6.	NPK + Mn (S)	12·02	24·22	8·35	15·95
7.	NPK + Mo (S)	11·72	23·02	11·10	18·20
8.	NPK + All (S)	14·12	26·37	10·65	18·45
9.	Sp. (S)	11·95	24·05	11·55	19·30
10.	NPK + Cu (F)	12·85	22·77	12·45	22·35
11.	NPK + Zn (F)	13·00	22·75	12·20	23·45
12.	NPK + B (F)	12·47	23·52	8·85	12·55
13.	NPK + Mn (F)	11·85	24·40	10·35	20·60
14.	NPK + Mo (F)	12·72	26·02	10·95	19·95
15.	NPK + All (F)	13·25	22·75	8·80	18·30

TABLE 3

Effect of trace element application on nitrogen and phosphorus in wheat plants (Patiala)

S. No.	Treatment symbol	Nitrogen %		Phosphorus %	
		Straw	Grain	Straw	Grain
1.	C	0·43	2·06	0·199	0·569
2.	NPK	0·64	2·06	0·212	0·595
3.	NPK + Cu (S)	0·62	2·04	0·144	0·399
4.	NPK + Zn (S)	0·44	1·88	0·153	0·518
5.	NPK + B (S)	0·56	1·81	0·153	0·459
6.	NPK + Mn (S)	0·62	1·95	0·212	0·561
7.	NPK + Mo (S)	0·48	2·40	0·127	0·246
8.	NPK + All (S)	0·57	3·08	0·153	0·603
9.	Sp. (S)	0·60	1·87	0·212	0·476
10.	NPK + Cu (F)	0·58	2·12	0·229	0·535
11.	NPK + Zn (F)	0·48	2·14	0·127	0·500
12.	NPK + B (F)	0·66	2·06	0·187	0·425
13.	NPK + Mn (F)	0·60	1·98	0·238	0·425
14.	NPK + Mo (F)	0·60	2·08	0·187	0·442
15.	NPK + All (F)	0·60	1·92	0·188	0·442

TABLE 4

Effect of trace element application on nitrogen and Phosphorus in wheat plant (Pant Nagar)

S. No.	Treatment symbol	Nitrogen %		Phosphorus %	
		Straw	Grain	Straw	Grain
1.	C	0·35	1·84	0·119	0·408
2.	NPK	0·38	1·84	0·120	0·391
3.	NPK + Cu (S)	0·36	1·96	0·102	0·348
4.	NPK + Zn (S)	0·40	1·96	0·119	0·433
5.	NPK + B (S)	0·40	2·06	0·119	0·459
6.	NPK + Mn (S)	0·50	2·18	0·178	0·408
7.	NPK + Mo (S)	0·40	2·10	0·195	0·450
8.	NPK + All (S)	0·35	1·98	0·161	0·459
9.	Sp. (S)	0·40	1·92	0·238	0·459
10.	NPK + Cu (F)	0·38	2·07	0·178	0·425
11.	NPK + Zn (F)	0·43	2·27	0·144	0·408
12.	NPK + B (F)	0·32	2·12	0·153	0·323
13.	NPK + Mn (F)	0·36	2·00	0·229	0·391
14.	NPK + Mo (F)	0·43	1·97	0·204	0·238
15.	NPK + All (F)	0·40	1·96	0·212	0·408

TABLE 5

Copper and Zinc content in wheat plants (Patiala)

S. No.	Treatment symbol	Copper in ppm		Zinc in ppm	
		Straw	Grain	Straw	Grain
1.	C	13·33	2·00	31·20	48·6
2.	NPK	13·00	2·50	27·50	43·5
3.	NPK + Cu (S)	9·17	4·25	27·60	37·2
4.	NPK + Zn (S)	10·42	2·00	28·00	42·0
5.	NPK + B (S)	6·46	6·75	19·80	39·6
6.	NPK + Mn (S)	7·08	11·25	27·20	51·9
7.	NPK + Mo (S)	11·25	4·50	14·12	36·6
8.	NPK + All (S)	8·75	7·37	29·72	64·2
9.	Sp. (S)	11·89	13·25	25·52	48·6
10.	NPK + Cu (F)	14·72	5·25	37·20	49·5
11.	NPK + Zn (F)	5·83	2·75	32·00	60·9
12.	NPK + B (F)	9·17	2·00	23·00	53·1
13.	NPK + Mn (F)	8·89	1·50	12·20	37·5
14.	NPK + Mo (F)	10·00	2·25	7·60	40·5
15.	NPK + All (F)	17·22	4·00	44·00	58·5

S. No.	Treatment symbol	Copper in ppm		Zinc in ppm	
		Straw	Grain	Straw	Grain
1.	C	8·61	6·00	30·40	49·5
2.	NPK	7·50	4·00	52·00	48·9
3.	NPK + Cu (S)	13·47	8·25	23·40	51·3
4.	NPK + Zn (S)	14·44	5·75	26·00	68·4
5.	NPK + B (S)	12·50	12·50	15·40	78·0
6.	NPK + Mn (S)	5·67	4·00	51·60	58·2
7.	NPK + Mo (S)	7·14	3·50	15·60	54·0
8.	NPK + All (S)	5·88	4·25	76·00	52·8
9.	Sp. (S)	5·46	10·25	74·00	48·6
10.	NPK + Cu (F)	6·30	4·75	70·00	36·0
11.	NPK + Zn (F)	10·08	5·50	21·60	80·7
12.	NPK + B (F)	5·88	3·75	14·00	43·2
13.	NPK + Mn (F)	5·67	8·00	60·00	53·4
14.	NPK + Mo (F)	7·14	9·00	92·80	54·6
15.	NPK + All (F)	8·40	5·50	32·40	61·5

Results and Discussion

(i) Effect of trace element application on the yield of wheat :

Data in table 2 show that at Patiala Cu and Zn were beneficial in increasing grain yield when applied directly to the soil or as foliar spray. Response to the application of all micronutrients in combination was superior to the effect of Cu or Zn alone, especially when applied to the soil. At Pant Nagar Cu and Zn were beneficial as foliar sprays. The addition of all micronutrients in combination was found harmful under soil dressing or foliar spray. Soil application of spartin was ineffective both at Patiala and Pant Nagar.

The data reveal the importance of soil reaction in determining the response to micronutrients in relation to the method of application. Soil application of microelements slightly augmented the grain yield under slightly alkaline soil conditions, while the responsiveness of the above elements as foliar spray was evident under the acidic soil conditions of Pant Nagar.

Soil application of Mn and B and foliar spray of Mn and Zn were found beneficial for wheat grain yield (Koraddi and Seth[19]). Gautam *et al*[11] reported that soil application of Zn and other microelements (Cu, Mn, B, Mo, Fe and Mg) was superior as compared to foliar spray.

(ii) Effect on the uptake of nitrogen :

A study of table 3 shows that the uptake of nitrogen in straw was depressed appreciably on soil dressing with Zn, Mo and on foliar spray of Zn in Patiala soils. The effect of other treatments was less marked. On the other hand, the application of trace elements to Pant Nagar soils improved straw nitrogen in many cases (Table 4). The effect of Mn applied to the soil was most marked. On foliar spray Zn and Mo were found to exert some beneficial effect.

The uptake of nitrogen by grains was not markedly affected in general, in experiments conducted at Patiala. However, application of Mo to this soil

resulted in an increase in grain nitrogen. The soil dressing of Zn, B and spartin slightly lowered the nitrogen content of grains. Under the acidic soil conditions of Pant Nagar, foliar spray of Zn markedly improved N content of the grains. Soil dressing with B, Mn and Mo and foliar fertilization with B, Cu and Mn caused slight improvement in grain nitrogen content.

Koraddi and Seth[19] reported that application of micro-nutrients (Mn, Zn, B and Cu) except Zn applied to the soil increased the N content of wheat grains. In the same year Govindarajan and Gopalarao[13] reported that Mn fertilization did not improved N content of wheat plants. Andrushchenke[2] revealed that the effectiveness of trace elements in increasing NO_3 accumulation in spring wheat decreased in the order of Zn, B, Mo, Cu and Mn.

(iii) *Effect on the uptake of phosphorus* :

Foliar spray of Cu or Mn alone improved P uptake in straw at Patiala. Soil application of any single or, all the micro-nutrients in combination did not result in any such beneficial effect. At Pant Nagar, soil dressing of Cu considerably depressed P uptake in straw whereas Zn or B did not appear to effect P uptake in either way. All other treatments improved P content of straw.

Soil dressing with Cu and foliar spray of B or Mo had depressive effect on P uptake in grains at Pant Nagar. The remaining treatments enhanced P uptake by grains. At Patiala all treatments, except soil application of all micro-nutrients in combination depressed P content in grains. In slightly acidic soils of Pant Nagar, in general, the phosphorus content of the straw as well as of grains had improved under all treatments. In contrast at Patiala at most of the treatments had depressive effect on the uptake of P in plants. The increased P uptake under slightly acidic soil conditions may be related to the existence of physico-chemical conditions conducive for the release of fixed P. In slightly alkaline soils, the deficiency of mobile P becomes a limiting factor in the uptake of phosphorus.

Mo and Cu application to acidic soils improved P uptake in plants (Zhiznevskaya[28]). Zn and B have also favourable effect on P absorption by plants (Baroccio[4]). Kastori and Saric[15] showed that P uptake varied greatly at low concentrations of trace elements applied whereas higher concentrations they (Cu, Zn, B and Mn) favourably affected P uptake in plants. Gopalkrishnan[12] and Forster[8] observed that Cu application decreased P content of plants.

(iv) *Uptake of copper in relation to nitrogen and that of Zinc in relation to phosphorus* :

Data in tables 3 to 6 revealed that there was little variation in N content with change in Cu concentration in straw at Patiala (r = + 0·33) and Pant Nagar (r = 0·42). Similarly, relationship between Cu and N content of the grains was found non-significant. An examination of the data of some of the individual treatments would however reveal the existence of such a relationship e. g., soil application of Mo increased nitrogen along with Cu content of grains at Patiala.

A study of tables 3 to 6 indicated no significant relationship between Zn and P uptake in straw of both the varieties of wheat grown at Patiala (r = + 0·13) and Pant Nagar (r = + 0·448). In the grains at Patiala correlation coefficient (r = + 0·506) was significant at 5% level. Similar data for grains at Pant Nagar was not found statistically significant (r = + 0·220).

Zinc and P applied at moderate rates had a positive interaction between them in the plant tissue (Luders[20] and Fedorenko[6]). Tortini and Morini[9] observed that an application of moderate quantity of Zn (20 ppm) not only eliminated the

depressive effect of heavy P fertilization (300 to 1000 ppm) but also improved the uptake of Zn with P content in tomato and spinach.

Acknowledgements

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